Ruling Narrowly: Learning and Law Creation*

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Abstract

I develop a dynamic model of law creation in which the court is uncertain about the ideal rule. The court learns about the ideal rule through the cases it hears, which are in turn the result of rational choices of agents responding to the court’s previous decisions. Learning requires experimentation, and since agents choose optimally, learning is only possible experimentation is incentive compatible for the agent. The court provides incentives to the agent by setting penalties, and writing opinions that commit the court to sanctioning or punishing various actions.

The model generates several predictions. First, the efficacy of opinion writing is asymmetric - the court has an incentive to write broad permissive opinions, but no corresponding incentive to write broad restrictive opinions. Second, the court’s learning is inefficient - it does not induce learning that minimizes the expected future cost of uncertainty. Instead, the court will induce experimentation that increases the likelihood that it can amend its permissive opinion - since this policy tool is more efficacious. Third, since the court cannot always amend an opinion, it has a incentive to preemptively write broad opinions.

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1 Introduction

Courts play a central role in settling disputes between litigants. In resolving most disputes, the role of the court is to uncover the relevant facts in order to correctly apply an existing legal rule. In these cases, the court’s behavior is quite mechanical - it simply applies existing law to the case before it. However, in some cases, the existing law may be silent or unclear about how the disputed ought to be settled. In such cases the common law tradition envisions an additional, and often controversial, role for the court - to extend the law in way that allows it to resolve the issue at hand. It is this process of law creation that this paper seeks to address.

Judicial minimalism is the legal philosophy that judges ought to extend the law narrowly - only as much as is necessary to resolve the dispute at hand. “A minimalist settles the case before it, but it leaves many things undecided” (Sunstein, 2001). The minimalist approach has been advocated for by many current and former jurists, including Stephen Breyer, Ruth Bader Ginsburg, Sandra Day O’Connor and John Roberts. For example, in Ayotte v. Planned Parenthood of N. New England\(^1\), the court was presented with a challenge to a New Hampshire law that required parental notification before allowing teenagers access to abortion services. The court issued a narrow ruling that it would be unconstitutional to apply the New Hampshire law in the case of a medical emergency, whilst leaving open the broader question of the general rights of minors to procure an abortion. Writing for a unanimous court, Justice O’Connor opined that “we try to limit the solution to the problem.”

Minimalists cite many and varied reasons for their particular approach to law creation. One important motivation for the minimalists’ cautious approach is to minimize the likelihood that the court will make mistakes that it cannot easily undo. For example, in City of Ontario v. Quon\(^2\), Justice Kennedy’s opinion asserted that: “Prudence counsels caution before the facts in the instant case are used to establish far-reaching premises... A broad holding ...

\(^2\) City of Ontario v Quon, 560 U.S. ___ (2010)
might have implications for future cases that cannot be predicted.” Similarly, at his Senate confirmation hearing, Justice Alito stated:

“I think that my philosophy of the way I approached issues is to try to make sure that I get right what I decide. And that counsels in favor of not trying to do too much, not trying to decide questions that are too broad, not trying to decide questions that don’t have to be decided, and not going to broader grounds for a decision when a narrower ground is available.”

Minimalists also argue that the court ought not usurp the democratic prerogative of the elected branches by “legislating from the bench”. Instead, the court should “see itself as part of a system of democratic deliberation” by allowing a “continued space for democratic reflection from Congress and the states” (Sunstein, 2001). A similar argument contends that the court ought not commit itself to a particular theoretical approach to the law, given heterogeneous beliefs and the changing nature of societal values over time. “Minimalists believe that a free society makes it possible for people to agree when agreement is necessary, and unnecessary for people to agree when agreement is impossible.” (Sunstein, 2006)

The minimalist approach to adjudication is, of course, not without its own problems. Whilst the minimalist court’s “epistemic humility” may reduce the costs associated with adopting sub-optimal rules, it perpetuates the existing uncertainty surrounding the law, which is itself costly. Even justices who typically advocate for the minimalist approach are cognizant of the significant costs that uncertainty places on future decision making. For example, in Blakely v Washington 3, the majority held that the State of Washington’s criminal sentencing system, which gave judges the ability to increase sentences based on their own determination of facts, violated the Sixth Amendment right to trial by jury. However, the majority opinion did not address the constitutionality of the Federal Sentencing Guidelines, which had many similarities to the Washington law. In a dissenting opinion, Justice Breyer argued: “But this case affects tens of thousands of criminal prosecutions... Federal prosecutors will proceed with those prosecutions subject to the risk that all defendants in those cases will have to be

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sentenced, perhaps tried, anew. Given this consequence and the need for certainty, I would not proceed further piecemeal.” The narrow approach “all but invites further challenges” (Smith, 2010). Indeed, the very first case that the Court heard in the following term - *United States v Booker*⁴ - dramatically changed the legal framework within which legal sentencing takes place at the Federal level (Siegel, 2005).

In this paper, I develop a model that captures the trade-off that the court faces between reducing uncertainty about the law on the one hand, and forgoing the opportunity to learn and thereby entrenching potentially sub-optimal rules, on the other. In so doing, I provide a framework with which to assess the merits of the minimalist approach. This paper focuses on the informational aspects of decision making. I ignore the democracy and heterogeneity based arguments for minimalism by considering a common value problem, where all similarly informed judges would make the same choices.

I model decision making within the context of a profit maximizing firm whose production generates a negative externality (such as water pollution) that imposes a harm on third parties. In the absence of regulation, the firm will generate more output than is socially efficient, since it fails to internalize the external cost it imposes upon others. The size of the external cost is unknown - although the court and firm share common beliefs about its value. As the court hears different cases, it observes whether the firm’s output (in that case) was inefficiently large or not, and updates its beliefs about the efficient output level. In this way, the court “learns” about what the ideal policy ought to be. (The learning process described has many parallels with the literature on price setting by a monopolist facing an unknown demand. For example, see Rothschild (1974) and Aghion et al. (1988).)

The court’s role is to regulate the firm’s behavior in order to implement the socially efficient outcome. The court does this by announcing a partial legal rule - which specifies thresholds for production below (above) which the firm will be definitely (definitely not) held liable - and a penalty for over-production. For example, the partial legal rule may definitely hold

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the firm liable whenever it produces a quantity in excess of 60, and definitely hold the firm not liable if it produces a quantity below 50. The rule is silent as to the liability status of a firm producing between 50 and 60 units of output - it will be held liable or not depending on the outcome of the court’s investigation, as explained above. In the uncertain region, the firm’s beliefs about the size of the efficient allocation will determine its assessment of the probability of being penalized.

In writing its opinion, the court can commit to holding the firm liable (or not) even at output levels where it is uncertain about what the ideal outcome ought to be. For example, the court’s belief may be that the efficient output level lies between 45 and 70. Then, in the above example, the announced rule commits the court to holding a firm that produces 50 units of output not liable even though the court believes with some probability that the efficient level is actually below 50. Similarly, it commits the court to find a firm producing 61 units liable, even though its beliefs suggest that the efficient level may actually be above this level. The lower threshold is permissive, since it establishes the region over which the firm may produce with impunity. By contrast, the upper threshold is restrictive, since it establishes the region over which the firm will be punished for sure.

A rule is narrow if it commits to holding the firm liable (not liable) only in cases where it puts probability one on the firm’s output being inefficiently high (low). A rule is broad if it finds the firm definitely liable or definitely not liable even in cases where the common beliefs imply some uncertainty. (Such a rule is broad, because it establishes a rule even for cases of the sort that the court has not heard and had an opportunity to learn about, and, as such, potentially entrenches costly errors.) The effect of broad rules is to create a wedge between the probability that the firm expects that it ought to be penalized - as implied by its common beliefs - and the probability that it will actually be penalized. In so doing, the court can create incentives for the firm to produce more or less than it might otherwise do.

The model’s focus on the regulation of a polluting firm is purely to give context to the analysis. The model could be applied equally well to any other situation in which the court
seeks to regulate the behavior of agents whose incentives are misaligned. For example, the framework presented could accommodate a model of search and seizure jurisprudence, in which the court seeks to regulate the conduct of the police - who have an incentive to conduct more invasive searches than is socially desirable, given individuals’ privacy interests. Similarly, the framework could accommodate a model of separation of powers - in which the court seeks to regulate the activity of the executive branch (for example) or the federal government - who may have an incentive to extend their influence into matters that are properly the realm of other branches and/or tiers of government.

An important ingredient in my model is that the firm is assumed to rationally respond to the court’s prior rulings and to other available information. (This assumption stands in contrast to other models of law creation, including Baker and Mezzetti (2010) and Niblett (2010), in which agent behavior is unmodeled and often assumed to be drawn randomly from some legal-rule-invariant distribution.) The assumption that agents are responsive to the court’s rulings becomes especially important given that many common law countries require that courts only adjudicate actual controversies. The court cannot simply expound on the law if there is no actual dispute between parties that requires resolution. It cannot adjudicate hypothetical cases. This procedural constraint, along with the fact that the firm chooses policy rationally, has two important implications. First, the court may not be able to learn about the validity of a particular type of behavior if agents never choose to behave in that way. The court can learn about the ideal rule only insofar as it can provide incentives for the firm to experiment by choosing an output level in the uncertain region. The court does not learn by experimenting - rather it learns by having others experiment - and to this extent, its learning is limited by its ability to make experimentation incentive compatible, which in turn depends upon the legal rule and beliefs. Hence, in adjudicating a case, the court must be cognizant of the effect of its current decision on its future ability to learn. Second, the ability of the court to modify the law in the future is limited to the extent that a case arises in the future that requires the court to modify its rule. This may create an incentive for
the court to preemptively write broader opinions than it otherwise might, just in case the
opportunity to revise the rule does not arise soon enough in the future.

A second important feature of the model, that has consequences related to those above, is the
assumption that common law courts are bound by the doctrine of *stare decisis* - or respect
for precedent. Adherence to precedent implies that the court will mechanically dispose of
cases that are already governed by an existing rule. This further prevents learning. The
law only progresses when the court can provide incentives for the firm to experiment in the
uncertain region of the law.

The model makes several predictions about the nature of law creation. First, the model
demonstrates that there is an asymmetry in the efficacy of permissive rules *vis a vis* restrictive
rules, in affecting the firm’s output choice. This reflects the inherent asymmetry in the
model, in which the unregulated firm deviates systematically from the efficient level, by
over-producing. Making the rule more permissive then will cause the firm to weakly increase
its output. By contrast - making the rule more restrictive can cause the firm’s choice to both
increase or decrease. (If the penalty is not too high, as the rule becomes more restrictive, the
firm may prefer to simply over-produce and bear the penalty for sure, rather than produce a
lower level of output at which it may still be penalized.) The model predicts that the court
ought to write broader permissive opinions and narrow restrictive opinions. For example,
when ruling on the legality of police action, the court ought to write broader opinions in
cases where it finds the police action to be appropriate, but narrow opinions when it seeks
to limit police power. The implication for minimalism is a doctrine is thus divided. Narrow
opinions are ideal when the court seeks to restrict the behavior of agent, but broad opinions
are preferred when the court seeks to encourage that behavior.

Second, the model predicts that the court will use its opinion to entice the firm to experiment
- but that the experimentation that it induces generates an inefficiently low level of learning.
(One way to think about the court inducing experimentation is that courts often signal
to future potential litigants about the sorts of cases that it would like to hear.) Efficient
learning occurs when the court experiments at a level that minimizes the expected future cost of uncertainty. (I will define this more precisely in Section 5.) In my simple model, efficient learning requires the firm to experiment at the mean of the uncertain region. This makes it equally likely that the experimental level will be found to be acceptable or not, and hence equally likely that the court will be able to revise its lower and upper opinions. However, since the court’s policy tools are not equally efficacious, the court would rather decide a case that allowed it to revise its lower rule than its upper rule. This creates an incentive for the court to entice the firm to experiment at a level at that is more likely to be found acceptable, than not. The court would rather learn less (in the sense that the expected future costs of uncertainty are larger), but acquire information that it can use more effectively to implement the efficient allocation in the future.

This model contributes to an emerging literature on judicial decision making and law creation. The two papers most similar to this are Fox and Vanberg (2011) and Baker and Mezzetti (2010). Fox and Vanberg (2011) consider a court that has determined that the legislature’s current policy is unconstitutional, but is unsure of what the ideal law ought to be. They show that by writing a broad opinion, the court can force the legislature to experiment in a region of the policy space where it can learn more efficiently about the ideal law. Fox and Vanberg (2011) consider a one-sided model of law creation (in which the court either determines a policy to be definitely unconstitutional or not) - and so their model does not display the asymmetries that feature importantly in this model. Moreover, they consider a context in which the court’s opinion is the only tool that it can use to affect the legislature’s policy - there is no other penalty for non-compliance (such as fine) - and this implies a greater need for the court to write broad opinions. By providing the court with additional policy tools, my model ‘stacks the deck’ in favor of minimalism, but nevertheless finds that the court ought to write broad opinions in certain cases.

Baker and Mezzetti (2010) model a resource constrained court that must, in each period, determine how broadly to construe the outcomes of previous cases. They show in an infinite
horizon model that the law will always converge, and provide conditions under which it converges to the efficient level. Whilst their analysis is two-sided, the decisions of the agents whose behavior is being regulated by the court are unresponsive to the court’s policy - cases are simply drawn from a fixed distribution. Although the analysis in this paper is limited to two-periods, it demonstrates the important limitations on court learning when agents are assumed to respond rationally to the court’s decisions. Indeed, contrary to Baker and Mezzetti (2010), this paper shows as that the amount of uncertainty diminishes, the court is increasingly limited in its ability to provide incentives for the firm to continue to experiment. When the degree of uncertainty is small enough, experimentation stops entirely.

Both these papers also make reduced form assumptions about the nature of judicial preferences. To my knowledge, this is the first model that fully endogenizes the court’s preferences, and which models the costs of uncertainty in a structural rather than reduced form way. As will become clear, this has important consequences for the sorts of conclusions that follow from the model.

These papers are the intersection of two strands of the literature on judicial politics. Previous studies of law creation have typically investigated the implications of heterogeneity (bias) in judicial preferences on the evolution of the common law. (See, for example, Gennaioli and Shleifer (2007), Niblett (2010), Ponzetto and Fernandez (2008) .) Gennaioli and Shleifer (2007), for example, provide foundations for the “Cardozo Theorem”, which states that the individual biases of judges tend to wash out as law is created piece-meal. These models typically do not involve any uncertainty about the ideal legal rule - there is no role for learning. A separate literature - most notably Clark and Kastellec (2010) and Beim (2012) - consider models of judicial learning. These papers focus on the Supreme Court’s decision to grant certiorari (or not) in a given case, based on signals from lower court decisions in related cases. A related literature focuses on the informational aspects of decision making, where a superior court cannot perfectly convey its ideal policy to inferior courts. (See, for example, De Mesquita and Stephenson (2002) and Cameron, Segal and Songer (2000).)
The remainder of this paper proceeds as follows: In section 2 I present the formal model. Section 3 analyzes the full information case and shows that the court implement the efficient allocation. Section 4 considers the firm’s decision under uncertainty, and Section 5 analyzes the court’s optimal choice. Section 6 presents some extensions.

2 Model

There is a profit maximizing firm that must choose the quantity to produce of a single good. The firm’s profit from producing \( q \) units of output is \( \pi(q) = \alpha q - \frac{1}{2} \beta q^2 \), where \( \alpha, \beta > 0 \). The profit maximizing level of output is \( q_H = \frac{\alpha}{\beta} \). The concavity of the profit function implies that the firm is averse to risk.

The firm’s production generates an externality that harms a third party. The external cost of \( q \) units of output is \( C(q) = \theta q \). In the presence of the negative externality, the unregulated firm will produce more output than is socially optimal, because it does not internalize the external cost of its production. The socially efficient level of output is \( q^* = \frac{\alpha - \theta}{\beta} \). To ease notation, I let \( S(\theta) = \frac{\alpha - \theta}{\beta} \) denote the efficient quantity when the marginal external cost \( \theta \), and let \( T(q) = \alpha - \beta q \) denote the marginal external cost for which \( q \) is the efficient output level. Obviously, \( S(\theta) = T^{-1}(\theta) \)

The size of the marginal external cost \( \theta \) is unknown. I assume that all agents share common beliefs about \( \theta \). For simplicity, suppose prior beliefs are uniformly distributed on the interval \([l_0, u_0] \), where \( 0 < l_0 < u_0 < \alpha \). These bounds imply that the efficient output level is positive \( (q^* \geq \frac{\alpha-l_0}{\beta} > 0) \) but less than the level that maximizes the firm’s profit \( (q^* \leq \frac{\alpha-l_0}{\beta} < \frac{\alpha}{\beta}) \). This implies a role for an efficiency minded court to regulate the firm’s behavior, and implement the socially efficient outcome.

The external party may bring a case before the court to seek compensation if it experiences harm. The court uses the \textit{liable if over-produce} rule, whereby it holds the firm liable for
damages if it finds that the firm has over-produced relative to the socially efficient output level. This is analogous to the Hand Rule\(^5\) that determines the standard of care in negligence cases.\(^6\) Since the external party is always harmed by the firm’s production, the court will always be adjudicating a genuine controversy between the parties. For simplicity of exposition, I assume that the external party brings a case before the court whenever it believes that there is a positive probability of being compensated. If the firm is held liable, it must pay a flat penalty \(F\). (The assumption that the fine is invariant to the level of output simplifies the analysis considerably, however I acknowledge that it is quite restrictive. In Section 6 I demonstrate that a proportional penalty schedule \(F = fq\) has similar features to the flat penalty, and argue that the qualitative implications of the simple model should extend to a general class of continuous and increasing penalty schedules.)

The law is described by a partial legal rule \((\lambda, \mu)\) that partitions the case-space into three equivalence classes. For any output \(q \leq \lambda\), the rule prescribes that the firm will not held liable (even if beliefs assign a positive probability to the chosen output being inefficiently large), and for any \(q > \mu\), the rule prescribes that the firm will definitely held liable (even if beliefs assign a positive probability to the chosen output being below the efficient level). Whenever the court encounters a case that is governed by the partial rule, it mechanically applies the rule and determines the firm to be liable or not, as required. (Alternatively, in a hierarchical court structure, lower courts are assumed to always mechanically apply the higher court’s rule once set, and the higher court will never agree to review such cases.) The partial rule is silent about the validity of choices in the region \(\lambda < q \leq \mu\). If a case arises in this region, the court fully investigates the case - it hears expert testimony, receives amicus briefs etc. - and perfectly learns whether the chosen output level was above of below the socially efficient level. Note that learning is only possible if a case arises in the ambiguous region, since

\(^5\)United States v. Carroll Towing Co. 159 F.2d 169 (2d. Cir. 1947).

\(^6\)Indeed, Judge Learned Hand proposed (in quite explicit mathematical terms) that the defendant be held liable for damages if the expected reduction in the injury from taking extra precaution outweighed the additional cost - i.e. if less than efficient precaution was taken. Judge Hand wrote: “Possibly it serves to bring this notion into relief to state it in algebraic terms: if the probability be called \(P\); the injury, \(L\); and the burden, \(B\); liability depends upon whether \(B\) is less than \(L\) multiplied by \(P\); i.e., whether \(B < PL\).”
outside this region, the court mechanically disposes of the case without investigation.

A latent assumption is that the court chooses rules that are monotonic - i.e. whenever the rule finds \( q \) to be acceptable, then it will also find any \( q' < q \) acceptable, and whenever it finds \( q \) unacceptable, it will also find any \( q' > q \) unacceptable. Given that the case space is uni-dimensional and ordered, it is natural to focus on monotonic rules of this sort. Moreover, the unidimensionality of the case space provides an ordering over policies that allows a natural way to classify rules as narrow or broad.

The court’s investigation is public, and so all players update their beliefs about \( \theta \) in the same way after observing the outcome of a particular case. Hence, beliefs are common. The posterior distribution of beliefs remains uniform, since the prior was uniform and the new information simply truncates the support of beliefs. As an illustration, suppose the court finds output \( q \) to be inefficiently large, which implies that \( T(q) = \alpha - \beta q < \theta \). Then, the posterior beliefs are given by:

\[
\Pr[\theta \leq x|\theta > T(q)] = \frac{\Pr[T(q) < \theta < x]}{\Pr[\theta > T(q)]} = \frac{x - T(q)}{u_0 - T(q)}
\]

and so \( \theta^{\text{post}} \sim U[T(q), u_0] \). If \( q \) were found to be acceptable, then \( \theta^{\text{post}} \sim U[l_0, T(q)] \). Since beliefs are always uniform, they are completely summarized by the extreme values of the support of the distribution. Hence, from herein, I will denote beliefs by a pair \((l, u)\) where \( l \leq u \).

In addition to updating beliefs, the court must also extend the partial rule in a way that is consistent with the outcome of the case. This is the process of law creation and it is asserted through the court’s opinion in a given case. For example, if the existing partial rule is \((\lambda_0, \mu_0)\) and the court finds output level \( q \) to be unacceptable, then the court must write
an opinion that extends the partial rule such that $\mu_1 \leq q$. This ensures that if an identical case were to arise in the future, the future court would adjudicate the case in the same way. (Note importantly that the disposition of the case restricts the sorts of opinions that the court may write. The court cannot simply act as a legislator choosing its ideal policy - it is constrained by the facts of the case that it is adjudicating. In this respect, my model implicitly uses the case-space approach developed in Kornhauser (1992b) and Kornhauser (1992a).) I assume that courts are always bound by the doctrine of *stare decisis* - so that the court cannot undo a rule, once set. The court can only create new law in the ambiguous region. In principle, when writing an opinion, the court can extend both sides of the partial legal rule. However, I restrict the court to only amending the side of the law that is necessary to justify the outcome in the case before it. In the above example, the court must revise $\mu$ so that it is at a level that is consistent with $q$ being unacceptable. However, it may not revise $\lambda$, since doing so can never explain the outcome of the case. (Obviously the court cannot revise up $\lambda$ such that $\lambda > q$, since this would make the ruling inconsistent. Whilst it is consistent to set $\lambda < q$, this merely says that output level $q$ is not definitely acceptable, which does not explain that it is in fact unacceptable.)

It is without loss of generality to assume $S(u) \leq \lambda \leq \mu \leq S(l)$. To see why, suppose the court sought to write a rule with $\lambda < S(u)$. Such a rule would guarantee that any output $q \leq \lambda$ will be found acceptable. But all agents know that the court will never discover a case $q < S(u)$ to be inefficiently large - since $S(u)$ is the efficient output level associated with the highest possibly value of the external cost $\theta$, given beliefs. The firm could behave as if $\lambda = S(u)$, and this would not affect its decision in any way. Hence, it is without loss of generality to assume $\lambda \geq S(u)$. (A similar argument holds for $\mu \leq S(l)$). The legal rule only begins to have bite when it creates a wedge between the expected penalty implied by the law and the expected penalty implied by beliefs. For example, if the court writes an opinion with $\lambda > S(u)$, then for $q \in (S(u), \lambda]$ the court commits to finding the firm not liable as a matter of law, even though it believes with positive probability that the firm is over-producing. I
say that an opinion is *narrow* if the court’s opinion sets \( \lambda = S(u) \) or \( \mu = S(l) \). On the other hand, an opinion is *broad* if the court’s opinion sets \( \lambda > S(u) \) or \( \mu < S(l) \). Broad opinions potentially enable the court to affect the firm’s choice by distorting the probability that it expects to be penalized.

To capture the dynamic process of learning, I analyze the choices of both the court and firm, over two periods. The timing of the game is as follows. At period 1, a case \( q_1 \) exogenously arises. The court learns and writes an opinion. This specifies the environment for the next period, which is a pair of beliefs \((l_1, u_1)\), a partial rule \((\lambda_1, \mu_1)\) and a penalty \(F_1\). (I assume that this is the court’s first opportunity to create law on this issue. Given the above discussion, this is equivalent assuming that the period 0 legal rule was purely narrow.) The second period environment must satisfy the following: If \( q_1 \) was deemed acceptable, then \( l_1 = l_0 \) and \( \mu_1 = \mu_0 \), whilst \( u_1 = T(q_1) \) and \( \lambda_1 \geq q_1 \). On the other hand, if \( q_1 \) is deemed unacceptable, then \( u_1 = u_0 \) and \( \lambda_1 = \lambda_0 \), whilst \( l_1 = T(q_1) \) and \( \mu_1 \leq q_1 \). At period 2, the firm optimally chooses its output level \( q_2 \), given this new environment. If \( q_2 \notin (\lambda_1, \mu_1) \), then the court mechanically disposes of the case according to the existing law. The court does not investigate the case, and there is no learning. The environment for the following period remains unchanged. If \( q_2 \in (\lambda_1, \mu_1) \), then the court investigates the case, learns and writes a new opinion (which satisfy the aforementioned requirements). This generates a new environment, which again is a pair of beliefs \((l_2, u_2)\) a partial rule \((\lambda_2, \mu_2)\), and a penalty \(F_2\). In period 3, the firm chooses its output and the game ends.

The court’s objective is to implement the output that minimizes the expected (utilitarian) social deadweight loss. If \( \theta \) were known, the social utility from producing the efficient level of output is \( \frac{1}{2\beta} (\alpha - \theta)^2 \). With uncertainty, the per-period net social loss from a given output
choice $q$ is:

$$
E_\theta \left[ \frac{1}{2\beta} (\alpha - \theta)^2 \right] - E_\theta \left[ (\alpha - \theta) q - \frac{1}{2} \beta q^2 \right] \\
= \frac{1}{2\beta} E_\theta \left[ (\alpha - \beta q - \theta)^2 \right] \\
= \frac{1}{2\beta} \text{Var} [\theta] + \frac{1}{2\beta} (T(q) - E[\theta])^2
$$

(1)

In each period, the court seeks to provide incentives for the firm to choose the output level that minimizes the sum of present and future expected social losses. Equation (1) demonstrates that two factors contribute to the social deadweight loss. The first factor is the presence of uncertainty, which causes the variance term to be positive. The second factor is any deviation of the chosen output from the \textit{ex ante} efficient level. Since the court issues its ruling after the firm has made its choice, it cannot reduce uncertainty in the current period - although by inducing the firm to experiment, it may cause uncertainty to decrease in the future. In the final period, since there is no further benefit from learning, the court has a strict incentive to implement the \textit{ex ante} efficient output. In the first period, however, it must trade off the benefit from choosing the efficient first period outcome against the benefits of learning in the optimal way for the future.

3 Full Information Benchmark

The model of law creation that I described in the previous section embeds a standard externality problem. A well known solution to the problem is to force the firm to internalize the externality by imposing a Pigovian tax equal to the marginal external cost. However, the court does not use this mechanism, and in this paper, has only a blunt tool (in the form of a flat penalty) to provide incentives to the firm. In this section, I verify that a court using the \textit{liable if over-produce} rule can implement the efficient allocation in a full information environment.
Suppose $\theta$ is commonly known. The optimal output level is $S(\theta) = \frac{\alpha - \theta}{\beta}$. The court sets a rule whereby the firm is liable if and only if $q > S(\theta)$. The firm chooses output $q$ to maximize its profit:

$$\pi_f = \begin{cases} 
\alpha q - \frac{1}{2} \beta q^2 & q \leq S(\theta) \\
\alpha q - \frac{1}{2} \beta q^2 - F & q > S(\theta)
\end{cases}$$

**Proposition 1.** Suppose $\theta$ is known and the legal rule holds the firm liable whenever $q > S(\theta)$. Then the firm will choose the efficient output level $q = \frac{\alpha - \theta}{\beta}$ whenever $F \geq \frac{\theta^2}{2\beta}$. If $F < \frac{\theta^2}{2\beta}$, then the firm will produce $q_H = \frac{\alpha}{\beta}$, which is inefficiently large.

Proposition 1 shows that in the complete information case, the court can induce the efficient output level using the liable if over-produce rule, as long as the penalty for over-production is not too low. Moreover, the size of this penalty need not be unreasonably large. Indeed, the ‘natural’ penalty - the penalty that fully compensates the external party in the event that the firm over-produces - is sufficient to entice the firm to choose the efficient output level. To see this, suppose the firm ignored the fine and over-produced anyway. The best deviation for the firm is to produce $\frac{\alpha}{\beta}$ since this maximizes its pre-penalty profit. The harm to the external party is $\frac{\alpha}{\beta} \theta$. Letting $F_N = \frac{\alpha}{\beta} \theta$, we have:

$$F_N = \frac{\alpha}{\beta} \theta = \frac{\theta^2}{2\beta} \cdot \frac{2\alpha}{\theta} > \frac{\theta^2}{2\beta}$$

since $\alpha > \theta$.

This efficiency of the liable if over-produce rule mirrors the efficiency of the negligence rule in the absence of contributory negligence (see Brown (1973) and Cooter, Kornhauser and Lane (1979) amongst others).
4 Firm’s Decision

In this section, I consider the firm’s optimal decision at each stage in the game with uncertainty. I assume that, in each period, the firm simply chooses the output that maximizes its expected profit in that period - ignoring the effect of its current decision on the environment it may face in the future. This assumption implies that the firm’s policy function is time independent - given identical beliefs and rules, the firm will make the same choice in both periods - which simplifies the analysis considerably. (To this extent, I omit time subscripts on the variables in this section.) The assumption is natural in situations where there is a long lived court and a sequence of short-lived firms, each of which makes a decision in only one period. In section 6, I extend the analysis to the case of a long lived firm that is strategic in its first period choice.

Suppose the agents have beliefs \((l, u)\) and the court has issued prior opinions \((\lambda, \mu)\) that are consistent (in the sense that \(l \leq T(\mu) \leq T(\lambda) \leq u\)). Then, the firm’s profit function is:

\[
\pi(q) = \begin{cases} 
\alpha q - \frac{1}{2} \beta q^2 & q \leq \lambda \\
\alpha q - \frac{1}{2} \beta q^2 - \Pr[\theta > T(q)] F & q \in (\lambda, \mu) \\
\alpha q - \frac{1}{2} \beta q^2 - F & q \geq \mu
\end{cases}
\]

where \(\Pr[\theta > T(q)] = \frac{u - T(q)}{u - l}\). The profit function exhibits discontinuities at \(q = \lambda\) whenever \(\lambda > S(u)\) and at \(q = u\) whenever \(\mu < S(l)\). The discontinuities reflect the wedge that broad opinions create between the expected penalty implied by the law and the expected penalty implied by beliefs. If \(\lambda > S(u)\), then court will not hold the firm liable for producing \(q \in (S(u), \lambda]\) even though there is a positive probability that the firm is producing above the efficient level. However, as soon as the firm produces slightly beyond \(\lambda\), it is no longer immune to the penalty. In fact, the probability of receiving the penalty jumps discontinuously from 0 to the level implied by beliefs at \(q = \lambda\). The probability of being penalized similarly jumps
discontinuously to one, when the firm produces slightly above $\mu$.

The firm’s marginal profit is:

$$\pi_2'(q) = \begin{cases} 
\alpha - \beta q & q < \lambda \\
\alpha - \beta q - \frac{\beta}{u-l}F & q \in (\lambda, \mu) \\
\alpha - \beta q & q > \mu 
\end{cases}$$

The marginal profit function is piece-wise linear and also has discontinuities at $q = \lambda$ and $q = \mu$. If the firm produces $q \leq \lambda$ or $q > \mu$, then it is either penalized for sure, or not at all - and so the marginal profit is unaffected by the penalty. However, when the firm produces $q \in (\lambda, \mu]$, then a small increase in output increases the probability of being penalized. Since beliefs are uniform, and the penalty is constant, this reduces the expected marginal profit by a constant amount.

Figure 1 illustrates the nature of the marginal profit function for arbitrarily chosen beliefs $(l, u)$, partial rule $(\lambda, \mu)$ and penalty $F$. The area $L_2$ indicates the amount that the firm’s expected profit falls when output increases beyond $\lambda$ due to the discontinuous jump in the probability of being penalized. Similarly, the area $cdef$ if the amount that the firm’s expected profit falls when output increases beyond $\mu$ (due to the probability of being penalized increasing discontinuously to one).
The diagram indicates that there are three candidate solutions for the optimal output - $q = \lambda$, $q = q_M$ and $q = \frac{\alpha}{\beta}$. (Note that even though $\pi'(\lambda) > 0$, the firm may not wish to increase output beyond $\lambda$ since expected profits fall discontinuously at this output level. Also, since $\pi'(\mu) < 0$, it cannot be optimal to produce at $q = \mu$.) If the firm increases output from $q = \lambda$ to $q = q_M$, then it loses $L_2$ in expected profits, but gains $G_2$. Similarly, if the firm increases output from $q = q_M$ to $q = \frac{\alpha}{\beta}$, it gains $G_1$ in profit but loses $L_1$. (More precisely, the firm loses parallelogram $cdef$ and $\Delta q_M\mu c$, and then gains $\Delta \mu d\frac{\alpha}{\beta}$. The net effect is summarized by the gain $G_1$ and loss $L_1$.) For the scenario illustrated in figure 1, the firm prefers $q_M$ to $\frac{\alpha}{\beta}$, since $L_1 > G_1$, and the firm prefers $\lambda$ to $q_M$, since $L_2 > G_2$. Hence the firm’s optimal choice is $q^* = \lambda$.

The following proposition summarizes the nature of the firm’s optimal output decision, given beliefs $(l, u)$, the partial rule $(\lambda, \mu)$, and the penalty $F$. (The full statement of the firm’s equilibrium policy is provided in Section 8.1 in the Appendix.)

**Proposition 2.** Let the environment be given by beliefs $(l, u)$, a partial rule $(\lambda, \mu)$ and a penalty $F$. There exist thresholds $\lambda, \hat{\mu}(\lambda), \overline{F} < F'(\lambda) < \overline{F}(\lambda)$, and $\hat{F}(\mu)$ such that:

1. If $\lambda > \lambda$ or $u < 2l$, then the firm will produce $q_H = \frac{\alpha}{\beta}$ whenever $F < F'$ and $q = \lambda$ whenever $F \geq F'$

2. If $\lambda \leq \lambda$ and $u \geq 2l$, then the firm will produce $q_H = \frac{\alpha}{\beta}$ whenever $F < \overline{F}$ and $q = \lambda$ whenever $F > \overline{F}(\lambda)$. If $F \in \left[\overline{F}, \overline{F}(\lambda)\right]$, the firm will produce $q_M(F) = \frac{\alpha}{\beta} - \frac{F}{u-l}$ if $q_M(F) \in (\lambda, \mu)$. Else it will produce $q = \mu$.

Proposition 2 states the firm will choose its ideal output $q_H = \frac{\alpha}{\beta}$ whenever the penalty is low enough, and will produce the safe output $q = \lambda$ (which is guaranteed to not attract penalty) when the penalty is high enough. For moderate penalties, the firm will experiment by choosing $q_M = \frac{\alpha}{\beta} - \frac{F}{u-l}$ if this is quantity is indeed in the ambiguous region $(\lambda, \mu)$, and choose $q = \mu$ otherwise. The intuition behind this result can be seen in figure 2 which
graphically represents the case of $\lambda \leq S \left(2\sqrt{l(u-l)}\right)$ and $\mu \geq S(2l) > S(u)$. When the penalty is $F_1$ the firm is indifferent between producing $q_M(F_1) = S(2l)$ and $q = \frac{a}{b}$ (since the gains $G_1$ from increasing output from $q_M$ to $\frac{a}{b}$ are exactly balanced by the losses $L_1$). For any $F < F_1$, the gains would be larger than the losses, so the firm would strictly prefer to produce $\frac{a}{b}$. Similarly, the firm prefers $q_M(F)$ to $\frac{a}{b}$ whenever $F > F_1$. By a similar argument, the firm is indifferent between producing $q = \lambda$ and $q = q_M(F_2)$ when the penalty is $F_2$ - since the gains $G_2$ from producing $q_M$ are exactly balanced by the losses $L_2$. The firm strictly prefers $q_M$ to $\lambda$ when $F < F_2$ and strictly prefers $\lambda$ when $F > F_2$. Hence, the firm produces $\frac{a}{b}$ when the penalty is low ($F < F_1$), it produces $\lambda$ when the penalty is high ($F > F_2$) and it produces $q_M(F)$ when the penalty is moderate ($F \in [F_1, F_2]$).

An important observation is that the firm’s supply function is discontinuous in the size of the penalty $F$. For example, as $F$ increases from zero, the firm’s output choice remains constant at $\frac{a}{b}$ until $F = F_1$, and then it jumps down to $q = S(2l)$. In fact, the firm’s output choice is unresponsive to the penalty whenever $F < F_1$ or $F > F_2$; it is only responsive when $F \in [F_1, F_2]$. It is also only in this region that the firm “experiments” - by choosing an output in the ambiguous region, where the probability of being penalized is uncertain.7

---

7In this section, I use the terms “uncertain” and “ambiguous” in a particular way. Uncertainty exists when the agents’ beliefs imply a non-trivial probability that the output chosen lies above the socially efficient level. Ambiguity exists when the legal rule does not prescribe definitely whether the output chosen will incur a penalty or not. Whenever there is ambiguity, there must also be uncertainty - however, the converse need
Experimentation is important, since learning is only possible when the firm experiments.

Intuitively the scope for experimentation is greatest when the firm has written the narrowest opinion - since the ambiguous region is . By Proposition 2, the firm experiments only when \( \lambda \leq \bar{\lambda} \) and if \( \mu \geq \hat{q}(\lambda) \). For opinions \((\lambda, \mu)\) satisfying these conditions, the region of experimentation is \([S \left(u - \sqrt{u^2 - T(\lambda)^2}\right), S(2l)]\) whenever \( \mu \geq S(2l) \) and \([S \left(u - \sqrt{u^2 - T(\lambda)^2}\right), \mu]\) when \( \mu < S(2l) \). (See Appendix for a derivation of these terms.) Hence an increase in \( \lambda \) (i.e. a broader lower opinion) strictly decreases the scope for experimentation, whilst a decrease in \( \mu \) (i.e. a broader upper opinion) weakly decreases the scope - and the effect is strict if \( \mu < S(2l) \).

The next corollary describes the firm’s optimal choice, and the scope for experimentation, if the legal rule is perfectly narrow (i.e. if \( \lambda = S(u) \) and \( \mu = S(l) \)).

**Corollary 1.** Suppose the legal rule is perfectly narrow so that \( \lambda = S(u) \) and \( \mu = S(l) \). The firm’s optimal output choice satisfies:

- If \( u < 2l \), then \( q^* = \frac{\alpha}{\beta} \) if \( F < \frac{1}{2\beta} u^2 \), and \( q^* = \lambda \) otherwise. There is no experimentation.

- If \( u \geq 2l \), then \( q^* = \frac{\alpha}{\beta} \) if \( F < 2l \frac{u-l}{\beta} \), \( q^* = \lambda \) if \( F > u \frac{u-l}{\beta} \) and \( q^* = q_M(F) = \frac{\alpha}{\beta} - \frac{F}{u-l} \) otherwise. The experimentation region is \([S(u), S(2l)]\).

Corollary 1 shows that the even with the narrowest possible rule, the court may not be able to induce the firm to experiment. Moreover, the scope for experimentation depends upon the relative size of the uncertain range. If \( u < 2l \) (i.e. if the region of uncertainty is small), then marginally increasing output in the uncertain region causes a relatively large increase in the probability of being penalized. This creates a disincentive for the firm to experiment. If the penalty is low enough, it will produce \( q_H = \frac{\alpha}{\beta} \). Otherwise, it will produce \( q = \lambda \) and not risk being penalized at all. Hence, as the amount of uncertainty narrows, the scope for not be true if the court’s opinions are broad enough.
the court to learn disappears completely. Once the extent of uncertainty is sufficiently small, learning stops completely.

5 Court’s Optimal Choice

In the previous section, I showed that the court cannot always to induce the firm to choose its desired level of output simply by varying the penalty. Consequently, court opinions that are purely narrow may result in inefficient social outcomes. The following lemma provides conditions under which narrow opinions and appropriately chosen penalties can induce the firm to choose the \textit{ex ante} optimal output.

\textbf{Lemma 1.} If the Court writes narrow opinions, $\lambda = S(u)$ and $\mu = S(l)$, it can induce the firm to choose the \textit{ex ante} optimal output $q = S\left(\frac{u+l}{2}\right)$ by setting $F = \frac{u^2-l^2}{2\beta}$, provided that beliefs satisfy: $u \geq 3l$. If this condition is not satisfied, then there is no penalty that the court can choose to induce optimal behavior.

Lemma 1 shows that a court that writes narrow opinions can only implement the \textit{ex ante} optimal output if uncertainty about costs is sufficiently large. If $u < 2l$, experimentation is not possible, so the firm will choose either $q = S(u)$ which is inefficiently low, or $q = \frac{2}{\beta}$ which is inefficiently high. If $2l \leq u < 3l$, then although experimentation is possible, the region of experimentation does not include the efficient allocation.

The next lemma shows that by writing broad opinions, the court may be able to implement outcomes that it would not have been able to implement using narrow opinions alone.

\textbf{Lemma 2.} Suppose the court can update its lower opinion $\lambda$. For any beliefs $(l,u)$, and for any feasible upper opinion $\mu \in [S(u), S(l)]$, there exists a pair $(\lambda, F)$ with $\lambda \in [S(u), \mu]$ that induces the firm to choose output $q$ if and only if $q \in [S(u), \mu] \cup \left\{\frac{2}{\beta}\right\}$. Moreover, any $q \in (\max \{S(u), S(2l)\}, \mu]$ can only be implemented by writing a broad opinion $\lambda = q$. 

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The court can induce the firm to choose any outcome in the ambiguous region, simply by choosing $\lambda$ and $F$ appropriately. Moreover, if the upper opinion $\mu$ is not too broad (i.e. if $\mu > S\left(\frac{u+l}{2}\right)$), then the court can always induce the firm to choose the ex ante optimum, by writing a broad lower opinion that targets the efficient level. Hence, as the opportunity to learn (or the benefit from learning) disappears, the court is best off writing a broad opinion to ensure that the efficient policy is chosen. The next lemma shows that the court’s ability to affect the firm’s choice using the upper opinion $\mu$ is far more restricted. Let $\hat{q}$ and $\hat{\mu}$ be as defined in Section 8.1.

**Lemma 3.** Suppose the court can update its upper opinion $\mu$ and consider any beliefs $(l, u)$. If $\lambda \in \left(S\left(2\sqrt{l(u-l)}\right), S(l)\right)$ or $u < 2l$, then the firm will either choose $q = \lambda$ or $q = \frac{u}{3}$, and this choice is independent of $\mu$. If $\lambda \in \left[S(u)\right]$, $S\left(2\sqrt{l(u-l)}\right)]$ and $u \geq 2l$, then there exists a pair $(\mu, F)$ that induces the firm to choose output $q$ if and only if $q \in [\hat{\mu}(\lambda), S(2l)] \cup \left\{\frac{u}{3}\right\}$. Moreover, any $q \in [\hat{\mu}(\lambda), \hat{q}(\lambda))$ can only be implemented by writing a broad opinion $\mu = q$.

Lemma 3 demonstrates that the court can affect the firm’s choice by manipulating $\mu$. However, the lemma also makes clear that the upper opinion $\mu$ is a much blunter instrument than the lower opinion $\lambda$. To see this, note that by manipulating $\lambda$, the court can entice the firm to choose any output in the range $q \in [S(u), \mu]$. The symmetric statement is not true for $\mu$ - the court cannot implement any $q \in [\lambda, S(l)]$ simply by manipulating $\mu$. Indeed output in the interval $(\max\{S(u), S(2l)\}, S(l)]$ is never implementable, and if the existing lower opinion is broad (i.e. $\lambda > S(u)$), then output in the region $(\lambda, \hat{\mu}(\lambda))$ is not implementable either. Finally, if the existing lower opinion is narrow (i.e. $\lambda = S(u)$), then any outcome that can be implemented by writing a broad opinion can also be implemented by writing a narrow opinion, and choosing $F$ appropriately. (This stands in contrast to the range of outcomes that only a broad lower opinion can implement, even when the upper opinion is narrow.)

The effect of opinion writing on the firm’s choice is asymmetric, and is a consequence of the
incentive for the firm to produce output at a level that deviates from the efficient level in a systematic way. Since the lower regulation $\lambda$ is permissive (in the sense that it provides a region of penalty free production), the firm will always find it desirable to produce at least $\lambda$ units of output. The court can then ensure that the firm does not produce beyond this level by setting the penalty for over-production arbitrarily high. The upper regulation $\mu$, on the other hand, is restrictive - it extends the region of guaranteed punishment. Unlike with $\lambda$, the firm will not always find it optimal to produce at most $\mu$ units of output - if the penalty is small enough, it will choose to produce $q_H$ and receive the penalty for sure. Moreover, since the firm’s supply function is discontinuous, increasing the penalty may cause the firm’s output to jump below $\mu$. Hence, the court cannot use the upper opinion, $\mu$, to target a desired output level in the way that it can use the lower opinion, $\lambda$.

5.1 Second Period Opinion

Suppose at the beginning of the second period, the firm chooses output level $q_2$. If $q_2 \leq \lambda_1$ or $q_2 > \mu_1$, then the court mechanically applies the existing rule. If $q_2 \in (\lambda_1, \mu_1]$, then the court must investigate the case. There are two scenarios to consider. In the first case, the court learns that a case $x$ is acceptable. It updates its beliefs so that $u_2 = T(q_2)$ (with $l$ unaffected so that $l_2 = l_1$) and writes an opinion $\lambda_2 \geq q_2$ (with $\mu_2 = \mu_1$ fixed). In the second scenario, the court learns that $q_2$ is unacceptable. It updates its beliefs so that $l_2 = T(q_2)$ (with $u_2 = u_1$ unaffected) and writes an opinion $\mu_2 \leq q_2$ (holding $\lambda_2 = \lambda_1$ fixed). The question of interest is where the court locates its opinions, $\lambda_2$ and $\mu_2$. Since the game ends after the second period, the court would ideally implement the ex ante optimum ($S\left(\frac{u_2 + l_2}{2}\right)$).

The following propositions outline the court’s optimal second period policy:

**Proposition 3.** Suppose the court is able to revise its lower opinion $\lambda$ in the second period.

1. If $\mu_2 \geq S\left(\frac{u_2 + l_2}{2}\right)$, then the court will implement the efficient output level $q^* = S\left(\frac{u_2 + l_2}{2}\right)$ by choosing either: (i) $\lambda_2 = S\left(\frac{u_2 + l_2}{2}\right)$ and $F$ sufficiently high ($F \geq \frac{(u_2 + l_2)^2}{2\beta}$ is usually
sufficient); or (ii) \( \lambda_2 \in \left[ S(u_2), S\left(\sqrt{u_2^2 - \left(\frac{u_2-l_2}{2}\right)^2}\right) \right] \) and \( F = \frac{u_2^2 - l_2^2}{2\beta} \) provided \( u_2 \geq 3l_2 \).

2. If \( \mu_2 < S\left(\frac{u_2+l_2}{2}\right) \), then the efficient output cannot be implemented. The court will implement the second best outcome \( q = \mu_1 \) by choosing (i) \( \lambda_2 = \mu_2 \) and choosing \( F_2 \geq \frac{1}{2\beta} T(\mu_2)^2 \); (ii) \( \lambda_2 \leq S\left(\sqrt{u_2^2 - (u_2 - T(\mu_2))^2}\right) \) and \( F_2 \in \left[ \frac{u_2-l_2}{2\beta}, T(\mu_2) \right] \) provided that \( \mu_2 \leq S(2l_2) \); or (iii) \( \lambda_2 \in \left( \frac{u_2-l_2}{2\beta} T(\mu_2) \right)^{-1} \left[ \frac{u_2-l_2 T(\mu_2)^2}{u_2-l_2 T(\mu_2)}, S\left(\sqrt{u_2^2 - (u_2 - T(\mu_2))^2}\right) \right] \) and \( F_2 \in \left[ \frac{u_2-l_2}{2\beta} T(\mu_2)^2 \right]^{-1} \left[ \frac{u_2-l_2 T(\lambda_2)^2}{u_2-l_2 T(\mu_2)}, S\left(\sqrt{u_2^2 - (u_2 - T(\mu_2))^2}\right) \right] \).

If \( \mu_2 \geq S\left(\frac{u_2+l_2}{2}\right) \), then the existing upper opinion \( \mu_1 = \mu_2 \) was not written so broadly as to make it impossible to implement the ex ante optimal policy. This policy can be implemented in potentially one of two ways. First, the court can simply write a broad lower opinion, setting \( \lambda_2 = S\left(\frac{u_2+l_2}{2}\right) \). Then, if the penalty is high enough, the firm will optimally choose to produce \( \lambda_2 \) units of output. Second, the court can choose the penalty optimally (i.e. \( F_2 = \frac{u_2^2 - l_2^2}{2\beta} \)) so that the firm both experiments and chooses the ex ante optimal policy. This is only possible if the region of experimentation is broad enough. If the lower opinion is narrow, then it is sufficient for beliefs to satisfy \( u_2 \geq 3l_2 \). In fact, as long as the lower opinion is not too broad (i.e. \( \lambda_2 \leq S\left(\sqrt{u_2^2 - \left(\frac{u_2-l_2}{2}\right)^2}\right) \)), the condition on beliefs suffices. Note that choosing the latter option (where the firm is forced to experiment) requires a lower actual penalty (relative to the former case where the firm is definitely not penalized on the equilibrium path), since the positive marginal probability of being penalized disciplines the firm from over-producing.

If \( \mu_2 < S\left(\frac{u_2+l_2}{2}\right) \), then the existing upper opinion \( \mu_1 \) is too broad and so the court cannot implement the efficient policy. The court will implement the policy that is closest to the optimum - i.e. \( q = \mu_2 \). Again it can do this either in a brute-force way, by setting \( \lambda_2 = \mu_2 \) and choosing a penalty high enough, or by writing a narrower opinion and using the penalty to cause the firm to choose \( q = \mu_2 > \lambda_2 \). Proposition 3 provides conditions under which each of these options are available.
Proposition 4. Suppose the court is able to revise its upper opinion $\mu$ in the second period.

1. If $u_2 \geq 3l_2$ and $\lambda_2 \leq S \left( \sqrt{2 \frac{u_2+l_2}{2}} \right)$, then the court can implement the efficient output level. If $\lambda_2 \leq S \left( \frac{\sqrt{u_2^2 - \left(\frac{u_2-l_2}{2}\right)^2}}{2} \right)$, it can do this either by setting $F_2 = \frac{u_2^2-l_2^2}{2\beta}$ and $\mu_2 = S \left( \frac{u_2+l_2}{2} \right)$, or by setting $\mu_2 = S \left( \frac{u_2+l_2}{2} \right)$ and choosing $F_2 \in \left[ \frac{1}{\beta} \frac{u_2+l_2}{2}^2, \frac{u_2^2-l_2^2}{2\beta} \right]$; and if $S \left( \sqrt{u_2^2 - \left(\frac{u_2-l_2}{2}\right)^2} \right) < \lambda_2 \leq S \left( \sqrt{2 \frac{u_2+l_2}{2}} \right)$, it can do this by setting $\mu_2 = S \left( \frac{u_2+l_2}{2} \right)$ and choosing $F_2 \in \left[ \frac{1}{\beta} \frac{u_2+l_2}{2}^2, \frac{1}{\beta} \left( T (\lambda_2)^2 - \left(\frac{u_2+l_2}{2}\right)^2 \right) \right]$.

2. If $u_2 < 3l_2$ or $\lambda_2 > S \left( \sqrt{2 \frac{u_2+l_2}{2}} \right)$, then the efficient output is not implementable. If $u_2 < 2l_2$ or $\lambda_2 > S \left( 2 \sqrt{l_2 (u_2-l_2)} \right)$, the court will implement $q = \lambda_2$ by choosing $F_2 \geq \frac{T(\lambda_2)^2}{2\beta}$; if $2l_2 \leq u_2 < 3l_2$ and $\lambda_2 < S \left( 2 \sqrt{l_2 (u_2-l_2)} \right)$, the court will implement $q = S \left( 2l_2 \right)$ by choosing $F_2 = \frac{2l_2(u_2-l_2)}{\beta}$ and $\mu_2 \in [S (2l_2), S (l_2)]$; and if $S \left( \sqrt{2 \frac{u_2+l_2}{2}} \right) < \lambda_2 < S \left( 2 \sqrt{l_2 (u_2-l_2)} \right)$ and $u_2 \geq 3l_2$, the court will either implement $q = \lambda_2$ by choosing $F$ large enough, or set $\mu = \hat{\mu} (\lambda)$ and $F = F_4 (\hat{\mu} (\lambda))$.

Proposition 4 shows that a broad upper opinion is only necessary for efficiency if the existing lower opinion $\lambda_1 = \lambda_2$ is too broad. If the $\lambda_2 = S (u_2)$ is narrow, then it is never strictly necessary to write a broad upper opinion to generate second period efficiency. By contrast, Proposition 3 demonstrates that a broad lower opinion may be necessary even if the existing upper opinion is narrow.

5.2 First Period Decision

The court’s second period policy is simply aimed at implementing the *ex ante* efficient outcome in the final period, taking beliefs as given. However, in the first period, the court can potentially learn (and hence reduce costly future uncertainty) by enticing the firm to experiment. Recall, the expected social loss from producing output $q$ is $L = Var [\theta] + \frac{1}{2\beta} \left( T(q) - E[\theta] \right)$. The first term captures the social loss that results from uncertainty, whilst the second term captures the loss from choosing an inefficient outcome (i.e. one that
deviates from the *ex ante* social optimum.) I say that the court learns optimally, if it entices
the firm to experiment in such a way that minimizes the expected second period social loss
from uncertainty.

**Lemma 4.** Efficient learning requires experimentation at \( q = S\left(\frac{u+l}{2}\right) \).

Lemma 4 prescribes that efficient learning takes place when the firm experiments at the mean
of the belief distribution. Since the cost of uncertainty is strictly convex (it is quadratic in
the size of the uncertain region \((u-l)\)), the court has a strict incentive to smooth (prob-
abilistically) the size of the uncertain region that results after the acceptability of the first
period output level is determined. Intuitively, if experimentation occurs at the mean of the
distribution, then learning is greatest in expectation, since the uncertain region is reduced
by a half, regardless of the outcome. Moreover, Lemma 4 shows that the there is no conflict
between the court’s desire to efficiently learn and implementing the first period socially ef-
ficient allocation. However, as we will see below, the asymmetry in second period outcomes
induces the court to choose an inefficiently low level of learning in the first period.

I assume that there are no existing opinions prior to the first period - which is equivalent
to assuming that the period 0 partial legal rule is narrowly constructed (i.e. \( \lambda_0 = S(u_0) \)
and \( \mu_0 = S(l_0) \)). To avoid having to model the firm’s time 0 decision, I assume that a
case exogenously arises in the first period. (This avoids needing to specify the firm’s beliefs
about the expected penalty in period 0 - at a time when the court has not indicated what
the penalty will be.) As in the second period - two possibilities arise - either the case is
found to be acceptable or not.

### 5.2.1 Revising from above \((\mu_1)\)

First consider the case where \( q_1 \) is found unacceptable. Let \((l_1, u_1)\) be the new beliefs (where
\( u_1 = u_0 \) and \( l_1 = T(q_1) \)) and \( \lambda_1 = \lambda_0 = S(u_1) \). The court can revise its upper opinion by
choosing any \( \mu_1 \in [\lambda_1, S(l_1)] \). The following proposition characterizes the court’s optimal policy in this environment.

**Proposition 5.** Suppose the court is able to revise its upper opinion \( \mu \) in the first period. If \( u_1 < 2l_1 \), then learning is not possible. The court will implement the static second-best outcome \( q = S(u_1) \) by setting \( F_1 \geq \frac{u_1^2}{2\beta} \) and choosing any \( \mu_1 \in [S(u_1), S(l_1)] \). If \( u_1 \geq 2l_1 \), then learning is possible. The court will choose a penalty \( F_1^* = \frac{u_1 - l_1}{\beta} T(x^*) \) which induces the firm to experiment by producing

\[
x^* = \begin{cases} 
S \left( \frac{8-2\sqrt{10}}{3} u_1 + \frac{2\sqrt{10}-5}{3} l_1 \right) & u_1 \geq \frac{11-2\sqrt{10}}{8-2\sqrt{10}} l_1 \\
S (2l_1) & u_1 \in \left[ 2l_1, \frac{11-2\sqrt{10}}{8-2\sqrt{10}} l_1 \right]
\end{cases}
\]

, and will write an opinion \( \mu_1 \in \left[ \frac{x^* + S(l_1)}{2}, S(l_1) \right] \).

Proposition 5 has several interesting features. First, it shows that there is no strict incentive for the court to write a broad upper opinion. Since the existing lower opinion is originally narrow (and this implies \( \hat{\mu} (\lambda_1) = \lambda_1 = \hat{q} (\lambda_1) \)), Lemma 3 implies that any output that can be implemented by writing a broad opinion \( \mu_1 < S(l_1) \), can also be implemented by writing a narrow opinion \( \mu_1 = S(l_1) \) and choosing the penalty that creates the desired level of experimentation. Since experimentation is desirable, the court will prefer a narrow opinion with experimentation and learning, to a broad opinion.

Second, Proposition 5 shows that - whilst the court will seek to generate experimentation whenever possible - it will not induce efficient learning, even though this is maximizes its first period payoff. (This follows since \( x^* < S \left( \frac{u_1 + l_1}{2} \right) \). Inefficient learning is a direct result of the asymmetry in the efficacy of the court’s upper and lower opinions. Efficient learning requires that the court experiment in the middle of the uncertain region, and this implies that the future court will find the experimental level appropriate or not with equal probability. In the former case, the second period court will be able to revise up its lower opinion \( \lambda \) and perfectly target the second period efficient allocation. By contrast, in the latter case, the
second period court will be able to revise down its upper opinion \( \mu \), but by Proposition 4, it will be unable to target the efficient allocation. (This is true regardless of period one beliefs, since in this case, \( l_2 = \frac{u_1+l_1}{2} \), and so \( u_2 < 2l_2 \).) Since the social loss in the latter case is much larger than in the former, the court has an incentive to experiment in a way that makes the latter case less likely to arise. Hence, the experimental level chosen in equilibrium will be below the mean of the uncertain region. The court faces a strict trade-off between efficient learning and \textit{ex post} second period efficiency. At the optimum, the court learns “less” - but is able to use the information it learns more effectively.

Proposition 5 shows that there is no strict incentive for the firm to write a broad upper opinion. However, somewhat broad opinions are permissible. This follows because the court is indifferent between rules that generate the same outcome. Since writing a slightly broad opinion does not change the firm’s incentive to experiment at the desired level, such a broad opinion is permissible. But note, it is not permissible for the court to write an opinion that is so broad as to constrain future policy making. (This is reflected in the restriction \( \mu \geq \frac{x^* + S(l)}{2} \).) If the game had a longer horizon, then the region in which the court has scope to write broad policies will shrink, as it is constrained by the desire to keep policy flexible in the future.

5.2.2 Revising from below \((\lambda_1)\)

Now consider the case where \( q_1 \) is found to be acceptable. Unlike the previous case, there is now a strict incentive for the court to write a broad opinion. To see why, as above, if the court is able to experiment, then there is a strict incentive to experiment and learn. But, as has been shown, if the court learns that the experimental level is too high, then the best it can do is to implement \( q = \lambda_1 \) in the following period. Since in that period, the court cannot revise up \( \lambda \) to target the efficient level (it only has the option to review the upper opinion \( \mu \)), then the court would benefit from having preemptively written a broad opinion \( \lambda_1 \) in the first period - anticipating the desire to implement an outcome closer to the efficient level,
in the second period. This captures the intuition presented in the introduction - that since the court can only revise opinions in the course of settling genuine disputes, it may write broad opinions as a preemptive tool, to hedge against the risk of not having an opportunity to revise its opinion in the future.

**Proposition 6.** Suppose the court is able to revise its lower opinion $\lambda$ in the first period. If $u_1 < 2l_1$, the learning is not possible, and the court will write a broad opinion $\lambda_1 = S \left( \frac{u_1 + l_1}{2} \right)$ and implement this by choosing $F_1 \geq \frac{1}{2\beta} \left( \frac{u_1 + l_1}{2} \right)^2$. If $u_1 \geq 2l_1$, learning is possible. The court will experiment at $y^\ast$ with $S(u_1) < y^\ast < S \left( \frac{u_1 + l_1}{2} \right)$ by choosing $F_1 = \frac{u_1 - l_1}{3} T(y^\ast)$ and write a broad opinion $\lambda_1 = S \left( \sqrt{u_1^2 - (u_1 - T(y^\ast))^2} \right)$.

The optimal policy $y^\ast$ is determined by the first order condition (4), which is presented in the proof. Propositions 5 and 6 exhibit many similar features. In both cases, the court induces the firm to experiment, and the experimental level is below the efficient first period level. Furthermore, this implies that there is inefficient learning. The court skews its experimentation in way that causes it to learn less, but which allows it use more effective policy instruments in the second period with greater probability. However, learning is more efficient when the court is able to revise its lower opinion $\lambda_1$ in the first period. Since the court writes a preemptively broad opinion, the *ex post* social loss that results in the case that the experimental level is found to be too high, is not as large. As such, the incentive for the court to skew the allocation away from this outcome is not as great.

As proposition 6 shows, a strict application of the minimalist approach can result in suboptimal outcomes, in which there is significant underproduction of the good. By ignoring the effect of its ruling on future firms’ decision making, the minimalist approach can fail to adequately perform the court’s role of providing incentives to agents in order to generate efficient outcomes.
6 Extensions

6.1 Functional Form

The analysis thus far has relied on specialized (and unrealistic) functional forms for the firm’s profit function, the cost function (for the external party), the nature of the penalty and beliefs. In the most part, these assumptions were purely for convenience, to keep the analysis tractable. For example, the choice of quadratic profit for the firm generated linear first order conditions, which simplified the analysis. In principle, any continuous, strictly concave function that has a local maximum would suffice. Similarly, the choice of a linear cost function for the external party generated a constant marginal cost. Again, in principle, any continuous and increasing cost function would suffice.

The assumption that beliefs were drawn from a uniform distribution ensured that the beliefs in each period were drawn from the same family of distributions. It is a well known result that the uniform distribution is the only distribution with the property that a posterior generated by truncating the support of the prior, conforms to the prior distribution. (***** cite***** ) Hence, the uniform beliefs assumption prevented the need to re-examine the firm’s choice for different classes of posterior beliefs. However, the conformity of prior and posterior beliefs was never crucial to the analysis. In principle, any continuous prior distribution $f(\theta)$ with support $(l_0, u_0)$ and this implies a posterior distribution $\frac{f(\theta)}{F(u_t)-F(l_t)}$ for posterior beliefs with support $(l_t, u_t)$.

The assumption of a flat rate penalty, however, is less benign. For example, it should be clear that with a proportional penalty schedule $F = fq$, the firm will reduce its output for any positive penalty (unlike in the above analysis where the firm continued to produce its most desired output $q_H = \frac{\alpha}{\beta}$ for small positive fine.) Nevertheless, even with a proportional penalty schedule, the equilibrium retains many of the same features as the equilibrium in the simple model above. The diagram below shows the firm’s marginal expected profit at different...
levels of output, for a given environment. (The marginal expected profit becomes steeper for $q \in (\lambda, \mu)$ because a marginal increase in output both increases both the probability that the firm will be penalized, and the size of the fine.) It should be clear from the diagram that the firm’s supply function will exhibit the same discontinuous behavior as in the simpler model, above. For $f$ small enough, the firm will produce $q_H = \frac{\alpha-f}{\beta}$ (which is obviously decreasing in $f$). At some threshold penalty level, the firm’s output drops from $q_H$ to $q_M = \frac{\alpha(u-l)+(\alpha-u)f}{\beta(u-l+2f)}$, which (analogous to $q_M$ in the above model) is decreasing in $f$. Finally, at some higher threshold penalty level, the firm’s output drops to $q = \lambda$, where it remains fixed - even if the penalty increases further.

The model was solved with a flat-penalty to keep the analysis tractable. Whilst the quantitative results will be different, since the firm’s supply function retains the same qualitative properties, I assert that the main insights of the paper will continue to hold if the court adopts a proportional fine. Indeed, the insights should be robust to any continuous and weakly increasing penalty schedule.
6.2 Strategic Firm

In the previous sections, I assumed that the firm was myopic in its first period output choice - it simply maximized its expected profit in that period, ignoring the effect of its current choice on the future environment and hence on future expected profits. This assumption may be valid in situations where the firm-like agent engages in the regulated activity for only a short period of time. Of course, in other situations, the assumption is less appealing.

Strategic considerations matter only in so far as experimentation today can affect the environment - and hence the profit making opportunities - that the firm will face in the future. Let $\Pi(q; \lambda, \mu, l, u)$ be the firm’s continuation value in the second period if it chooses and output level $q$ in the first period. It was noted in the previous section that there are often (but not always) a range of optimal policies that the court can implement in the second period. Importantly, these policies are not payoff equivalent from the perspective of the firm. For example, the court can induce the firm to choose some output level $q'$ by writing a broad opinion $\lambda = q'$ and setting the penalty high enough. In equilibrium the firm always complies, and so it never pays the penalty. By contrast, the court could induce the firm to experiment at $q'$ by choosing the penalty appropriately. In this case, the firm will be penalized with positive probability - and so it’s expected payoff is strictly lower. Consequently, the continuation value $\Pi$ depends upon the firm’s beliefs about the court’s future strategies.

For concreteness I will consider the case in which the firm believes that the court will always write a broad second period opinion to implement its desired outcome in the final period (i.e. the court will not induce experimentation in the final period). This approach has the benefit that such a strategy is always available (by Lemma 2) since the court can always implement $q = \lambda$ by choosing the penalty large enough. Whilst, the alternative assumption - that the court writes narrow opinions and induces experimentation wherever possible - is probably more appealing, the analysis is significantly complicated by the fact that such a strategy is not always available.
**Proposition 7.** The scope for first period experimentation is reduced when the firm behaves strategically.

Proposition 7 shows that the set of output levels that the court can induce the firm to experiment at is strictly smaller when the firm behaves strategically. The intuition is straightforward. If the firm experiments at a high output level, then this output level will most likely be found to be inefficiently large, and the court will induce the firm to choose a much lower output in the future. By contrast, if the firm experiments at a lower output level, then there is a greater likelihood that this output level will be found to be acceptable and that the court will consequently induce the firm to choose a higher output in the future.

The effect of strategic behavior of the firm on the court’s policy choice depends on whether the reduction in the scope for experimentation imposes a binding constraint upon the court or not. Since the court’s seeks to maximize the social welfare and this goal is independent of whether the firm behaves strategically or not. However, the court can only implement outcomes that are incentive compatible for the firm. As long as the optimal period 1 output level remains incentive compatible, the court will induce the firm to choose this output level. (Of course, with a strategic firm, the penalty that the court will use to induce this output will be lower. But this does not affect the court’s choice, since the court’s preference over penalties is purely instrumental.) Hence, the simplification to myopic firms in the main analysis was important only insofar as it affected the choice set from which the court could choose in the first period. The simplification does not affect in any way the logic of the main results presented in the previous sections.

7 Conclusion

Hi
8 Appendix

8.1 Full Statement of Proposition 2

For each $F \geq 0$, let $q_M (F) = \frac{a}{\beta} - \frac{F}{u-l} = S \left( \frac{\beta}{u-l} F \right)$. Define $F_1 = 2l \frac{u-l}{\beta}$, $F_2 (\lambda) = \frac{u-l}{\beta} \left( u - \sqrt{u^2 - T(\lambda)^2} \right)$, $F_3 (\lambda) = \frac{1}{2\beta} T(\lambda)^2$, $F_4 (\mu) = \frac{u-l}{2\beta} \frac{T(\mu)^2}{(T(\mu)-l)}$ and $F_5 (\lambda, \mu) = \frac{u-l}{\beta} \frac{\mu-\lambda}{\mu-S(u)} T \left( \frac{\mu+\lambda}{2} \right)$. Further, let $F_\lambda = \frac{u-l}{\beta} T(\lambda)$ and $F_\mu = \frac{u-l}{\beta} T(\mu)$ and let $\hat{q}(\lambda) = S \left( u - \sqrt{u^2 - T(\lambda)^2} \right)$ and $\hat{\mu}(\lambda) = S \left[ \frac{T(\lambda)^2 + T(\lambda) \sqrt{T(\lambda)^2 - 4l(u-l)}}{2(u-l)} \right]$.

Proposition (Proposition 2). If $\lambda \leq S \left( 2 \sqrt{l(u-l)} \right)$ and $u \geq 2l$, then

- if $\mu \geq S(2l)$, then $q^* = \begin{cases} \frac{a}{\beta} & F < F_1 \\ q_M (F) & F_1 \leq F \leq F_2 (\lambda) \\ \lambda & F > F_2 (\lambda) \end{cases}

- if $\hat{q}(\lambda) \leq \mu < S(2l)$, then $q^* = \begin{cases} \frac{a}{\beta} & F < F_4 (\mu) \\ \mu & F_4 (\mu) \leq F < F_\mu \\ q_M (F) & F_\mu \leq F \leq F_2 (\lambda) \\ \lambda & F > F_2 (\lambda) \end{cases}

- if $\hat{\mu}(\lambda) < \mu < \hat{q}(\lambda)$, then $q^* = \begin{cases} \frac{a}{\beta} & F < F_4 \\ \mu & F_4 (\mu) \leq F \leq F_5 (\lambda, \mu) \\ \lambda & F > F_5 (\lambda, \mu) \end{cases}

- if $\mu \leq \hat{\mu}(\lambda)$, then $q^* = \begin{cases} \frac{a}{\beta} & F < F_3 (\lambda) \\ \lambda & F \geq F_3 (\lambda) \end{cases}$

where $\hat{q}(\lambda) \geq \hat{\mu}(\lambda) \geq \lambda$, with strict inequality whenever $\lambda > S(u)$ and equality when $\lambda = S(u)$. 
• If $\lambda > S \left(2\sqrt{l(u-l)}\right)$ or $u < 2l$, then $q^* = \begin{cases} \frac{\alpha}{\beta} & F < F_3(\lambda) \\ \lambda & F \geq F_3(\lambda) \end{cases}$

8.2 Proofs of Propositions

**Proof of Proposition 1.** Suppose the court holds the firm liable whenever $q > \frac{\alpha - \theta}{\beta}$. The marginal profit is $\pi'(q) = \alpha - \beta q$ whenever $q \neq \frac{\alpha - \theta}{\beta}$, and there is a jump-down discontinuity at $q = \frac{\alpha - \theta}{\beta}$. Hence there are two candidate optimal output choices for the firm: $q = \frac{\alpha}{\beta}$ (which solves $\pi'(q) = 0$) and $q = \frac{\alpha - \theta}{\beta}$. Computing the profit in each case:

\[
\pi \left( \frac{\alpha - \theta}{\beta} \right) = \alpha \left( \frac{\alpha - \theta}{\beta} \right) - \frac{1}{2} \beta \left( \frac{\alpha - \theta}{\beta} \right)^2 = \frac{1}{2\beta} (\alpha^2 - \theta^2)
\]

\[
\pi \left( \frac{\alpha}{\beta} \right) = \alpha \left( \frac{\alpha}{\beta} \right) - \frac{1}{2} \beta \left( \frac{\alpha}{\beta} \right)^2 - F = \frac{\alpha^2}{2\beta} - F
\]

Then clearly, $q = \frac{\alpha - \theta}{\beta}$ is optimal whenever $F > \frac{\theta^2}{2\beta}$.

**Proof of Proposition 2.** The optimal output either occurs at a point of discontinuity (in the profit function) or at a level that causes $\pi'(q) = 0$. (The latter is guaranteed to be a local maximum since the marginal profit function is strictly decreasing wherever $\pi$ is continuous.) Hence, there are 4 candidate solutions: (i) $q = \lambda$, (ii) $q = \mu$, (iii) $q = q_M(F) = \frac{\alpha}{\beta} - \frac{F}{u-l}$, and (iv) $q = \frac{\alpha}{\beta}$. Note further that $q_M(F) = S \left( \frac{\beta}{u-l} F \right)$. Consistency requires that $q_M$ is only a solution if $\lambda < \frac{\alpha}{\beta} - \frac{F}{u-l} < \mu$ - or alternatively, that $F_\mu = \frac{\alpha - \beta \mu}{\beta} (u-l) < F < \frac{\alpha - \beta \lambda}{\beta} (u-l) = F_\lambda$.

The firm’s utility at each of the candidate levels of output are: $\pi(\lambda) = \alpha \lambda - \frac{1}{2} \beta \lambda^2$, $\pi(\mu) = \alpha \mu - \frac{1}{2} \beta \mu^2 - \frac{u-T(\mu)}{u-l} F$, $\pi \left( \frac{\alpha}{\beta} \right) = \frac{\alpha^2}{2\beta} - F$ and

\[
\pi(q_M) = \alpha S \left( \frac{\beta}{u-l} F \right) - \frac{1}{2} \beta \left[ S \left( \frac{\beta}{u-l} F \right) \right]^2 - \frac{u-T \left( S \left( \frac{\beta}{u-l} F \right) \right)}{u-l} F
\]

\[
= \frac{\alpha^2}{2\beta} + \frac{\beta}{2} \frac{F^2}{(u-l)^2} - \frac{u}{u-l} F
\]

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If \( q_M (F) < \mu \), then \( \pi (q_M (F)) > \pi (\mu) \). This follows since \( \pi \) is continuous over the interval \([q_M (F), \mu]\) and \( \pi' (q) < 0 \) over this interval. Hence, if \( q_M (F) \) is feasible/consistent, then \( q = \mu \) cannot be the optimizer.

First, ignore \( \mu \), and consider the firm’s optimal choice between \( \lambda, q_M (F) \) and \( \frac{\alpha}{\beta} \) (assuming that \( q_M (F) \) is feasible). (I refer to this as the baseline analysis.) \( q_M (F) \) is preferred to \( q = \frac{\alpha}{\beta} \) if:

\[
\frac{\alpha^2}{2\beta} + \frac{\beta}{2} \frac{F^2}{(u-l)^2} - \frac{u}{u-l} F \geq \frac{\alpha^2}{2\beta} - F
\]

\[
F \geq \frac{2l(u-l)}{\beta} = F_1
\]

Similarly \( q_M (F) \) is preferred to \( q = \lambda \) if:

\[
\frac{\alpha^2}{2\beta} + \frac{\beta}{2} \frac{F^2}{(u-l)^2} - \frac{u}{u-l} F \geq \alpha \lambda - \frac{1}{2} \beta \lambda^2
\]

\[
F \leq \frac{u-l}{\beta} \left( u - \sqrt{u^2 - T (\lambda)^2} \right) = F_2 (\lambda)
\]

There are two cases to consider. Suppose \( F_1 \leq F_2 \). Then \( q^* = \frac{\alpha}{\beta} \) whenever \( F < F_1 \), \( q^* = q_M (F) \) whenever \( F \in [F_1, F_2] \), and \( q^* = \lambda \) whenever \( F > F_2 (\lambda) \). (To see this, note that when \( F < F_1 \), \( \frac{\alpha}{\beta} \) is preferred to \( q_M (F) \) and \( q_M (F) \) is preferred to \( \lambda \) - so by transitivity, \( \frac{\alpha}{\beta} \) is the optimal output choice. A similar syllogism verifies the optimality of \( q_M (F) \) and \( \lambda \) in the remaining regions.) Suppose instead that \( F_1 > F_2 \). Then obviously \( q^* = \frac{\alpha}{\beta} \) for \( F < F_2 \) and \( q^* = \lambda \) for \( F > F_1 \). But for \( F \in [F_2, F_1] \), \( q_M (F) \) is dominated by both \( \lambda \) and \( \frac{\alpha}{\beta} \). In this case, \( \lambda \) is chosen if:

\[
\alpha \lambda - \frac{1}{2} \beta \lambda^2 \geq \frac{\alpha^2}{2\beta} - F
\]

\[
F \geq \frac{1}{2\beta} T (\lambda)^2 = F_3 (\lambda)
\]

Hence, if \( F_1 > F_2 \), then \( q^* = \frac{\alpha}{\beta} \) whenever \( F < F_3 (\lambda) \) and \( q^* = \lambda \) whenever \( F \geq F_3 (\lambda) \). (I
show below that $F_2 < F_3 < F_1$ whenever $F_2 < F_1$.)

Now, since $F_1$ is constant and $F_2$ is decreasing in $\lambda$, there must exist some threshold $\hat{\lambda}$ such that $F_1 \leq F_2$ whenever $\lambda \leq \hat{\lambda}$. This requires:

$$\frac{2l(u-l)}{\beta} \leq \frac{u-l}{\beta} \left( u - \sqrt{u^2 - T(\lambda)^2} \right)$$

$$\sqrt{u^2 - T(\lambda)^2} \leq u - 2l$$

$$\lambda \leq S \left( 2\sqrt{l(u-l)} \right) = \hat{\lambda}$$

provided that $u \geq 2l$. Clearly if $u < 2l$, then the inequality can never be satisfied. It can be easily verified that $F_2 < F_3 < F_1$ whenever $F_2 < F_1$. Suppose $F_2 < F_1$ which implies $T(\lambda)^2 < 4l(u-l)$. Then: $F_3 = \frac{1}{2\beta} T(\lambda)^2 < \frac{2l(u-l)}{\beta} = F_1$. To see that $F_2 > F_3$, suppose not. Then $F_2 \geq F_3$ implies:

$$\frac{u-l}{\beta} \left[ u - \sqrt{u^2 - T(\lambda)^2} \right] \geq \frac{T(\lambda)^2}{2\beta}$$

$$\frac{1}{2} \left[ \sqrt{u^2 - T(\lambda)^2} - (u-l) \right]^2 \geq \frac{1}{2} l^2$$

$$\frac{T(\lambda)^2}{2} \leq 0$$

which is a contradiction.

Now, introduce $\mu$ into the analysis. Suppose $\lambda \leq \hat{\lambda}$ so that $F_1 \leq F_2$ and $q_M(F)$ is possibly chosen over some range of $F$. It has been shown that $\mu$ is never chosen if $\mu > q_M(F)$. I showed above that $q_M(F)$ is only chosen if $F_1 \leq F \leq F_2$, and so the largest value of $q_M(F)$ that is ever chosen is $q_M(F_1) = S(2l)$. Hence, if $\mu > S(2l)$, $\mu$ is never chosen, and the above analysis holds. Similarly, the smallest value of $q_M(F)$ that is ever chosen is $q_M(F_2) = S \left( u - \sqrt{u^2 - T(\lambda)^2} \right) = \hat{q}(\lambda)$.
Suppose \( \hat{q}(\lambda) \leq \mu < S(2l) \). Then \( \mu \) is preferred to \( \frac{\alpha}{\beta} \) if:

\[
\alpha \mu - \frac{1}{2} \beta \mu^2 - \frac{u - T(\mu)}{u - l} F \geq \frac{\alpha^2}{2\beta} - F
\]

\[
F \geq \frac{u - l}{2\beta} \frac{T(\mu)^2}{l - \mu} = F_4(\mu)
\]

Note that, since \( \mu < S(2l) \), \( F_4(\mu) < F_\mu \). (To see this, note that \( T(\mu) > 2l \) and so \( F_4 = \frac{u - l}{2\beta} T(\mu) \frac{\mu^2}{l - \mu} = \frac{u - l}{2\beta} T(\mu) = F_\mu \).) Furthermore \( \hat{q}(\lambda) < \mu \) implies that \( F_\mu < F_2(\lambda) \). (To see this, note that \( \hat{q}(\lambda) = q_M(F_2) = S\left(\frac{\beta}{u - l} F_2\right) \). Then \( S\left(\frac{\beta}{u - l} F_2\right) < \mu \) implies \( F_2 > \frac{u - l}{2\beta} T(\mu) = F_\mu \).) This implies that, for \( F > F_\mu \), \( q_M(F) \) becomes feasible again, and the baseline analysis holds. Hence \( q^* = \frac{\alpha}{\beta} \) whenever \( F < F_4(\mu) \), \( q^* = \mu \) whenever \( F \in [F_4(\mu), F_\mu) \), \( q^* = q_M(F) \) whenever \( F \in [F_\mu, F_2(\lambda)] \) and \( q^* = \lambda \) whenever \( F > F_2(\lambda) \).

Now suppose \( \mu < \hat{q}(\lambda) \). This implies \( F_\mu > F_2(\lambda) \) and so \( q_M(F) \) is never chosen. As with the above analysis, the firm prefers \( \mu \) to \( \frac{\alpha}{\beta} \) if \( F \geq F_4(\mu) \). Similarly, \( \mu \) is preferred to \( \lambda \) if:

\[
\alpha \mu - \frac{1}{2} \beta \mu^2 - \frac{u - T(\mu)}{u - l} F \geq \alpha \lambda - \frac{1}{2} \beta \lambda^2
\]

\[
F \leq \frac{u - l}{2\beta} \frac{T(\lambda) - T(\mu)}{u - T(\mu)} \left[ l \frac{T(\mu)^2}{2} \right] = F_5(\lambda, \mu)
\]

Again, there are two cases to consider. Suppose \( F_4 \leq F_5 \). Then \( q^* = \frac{\alpha}{\beta} \) whenever \( F < F_4(\mu) \), \( q^* = \mu \) whenever \( F \in [F_4(\mu), F_5(\lambda, \mu)] \), and \( q^* = \lambda \) whenever \( F > F_5(\lambda, \mu) \). (To see this, note that when \( F < F_4 \), \( \frac{\alpha}{\beta} \) is preferred to \( \mu \) and \( \mu \) is preferred to \( \lambda \) - so by transitivity, \( \frac{\alpha}{\beta} \) is the optimal output choice. A similar syllogism verifies the optimality of \( \mu \) and \( \lambda \) in the remaining regions.) Suppose instead that \( F_4 > F_5 \). Then obviously \( q^* = \frac{\alpha}{\beta} \) when \( F < F_5 \) and \( q^* = \lambda \) when \( F > F_4 \). But for \( F \in [F_5, F_5] \), \( \mu \) is dominated by both \( \lambda \) and \( \frac{\alpha}{\beta} \). As above, \( \lambda \) is chosen if \( F \geq F_3(\lambda) \). Hence, if \( F_4 > F_5 \), then \( q^* = \frac{\alpha}{\beta} \) whenever \( F < F_3(\lambda) \) and \( q^* = \lambda \) whenever \( F \geq F_3(\lambda) \). (I verify below that \( F_5 < F_3 < F_4 \) whenever \( F_5 < F_4 \).)
It remains to describe conditions under which $F_5 \geq F_4$. This requires:

\[
\frac{u - l}{2\beta} \frac{T(\mu)^2}{T(\mu) - l} \leq \frac{u - l}{2\beta} \frac{T(\lambda)^2 - T(\mu)^2}{u - T(\mu)} \leq 0
\]

\[
(u - l) T(\mu)^2 - T(\lambda)^2 T(\mu) + lT(\lambda)^2 \leq 0
\]

\[
T(\mu) \leq \frac{T(\lambda)^2 + T(\lambda) \sqrt{T(\lambda)^2 - 4l(u - l)}}{2(u - l)}
\]

which implies $\mu \geq \hat{\mu}(\lambda) = S \left[ \frac{T(\lambda)^2 + T(\lambda) \sqrt{T(\lambda)^2 - 4l(u - l)}}{2(u - l)} \right]$. (Note that $\lambda \leq S \left( 2\sqrt{l(u - l)} \right)$ ensures that the expression under the square root is well defined.) Hence $F_4 \leq F_5$ whenever $\mu \geq \hat{\mu}(\lambda)$. Note that $\lambda \leq \hat{\mu}(\lambda) \leq \hat{q}(\lambda)$ and that the inequalities are strict whenever $\lambda > S(u)$. To see this, note that:

\[
T(\hat{\mu}(\lambda)) = \frac{T(\lambda)^2 + T(\lambda) \sqrt{(T(\lambda) - 2l)^2 - 4l(u - T(\lambda))}}{2(u - l)}
\]

\[
\leq \frac{T(\lambda)^2 + T(\lambda)(T(\lambda) - 2l)}{2(u - l)}
\]

\[
= T(\lambda) \frac{T(\lambda) - l}{u - l}
\]

and that $\frac{T(\lambda) - l}{u - l} \leq 1$, with strict inequality whenever $\lambda > S(u)$. Similarly: ******.

Suppose $\mu < \hat{\mu}(\lambda)$ so that $F_5 < F_4$. I must verify that $F_5 < F_3 < F_4$. Since $F_5 < F_4$, $T(\lambda)^2 < \frac{(u - l)}{[T(\mu) - l]} T(\mu)^2$, which implies that $F_3(\lambda) < F_4(\lambda)$. Suppose $F_3 \leq F_5$. Then $\frac{T(\lambda)^2}{2\beta} \leq \frac{u - l}{u - T(\mu)} T(\mu)^2$, which implies $T(\lambda)^2 \geq \frac{(u - l)}{[T(\mu) - l]} T(\mu)^2$, which is a contradiction.

Finally, if $u < 2l$, then $F_\lambda \leq F_1$. The firm must choose between $q = \lambda$ and $q = \frac{u}{\beta}$. By the above arguments, it was choose the former if $F \geq F_3(\lambda)$. 

**Proof of Lemma 1.** By Corollary 1, we know that experimentation is only possible with perfectly narrow opinions if $u \geq 2l$. (Of course, for broader rules, the scope for experimentation is even smaller.) The region of experimentation is $[S(u), S(2l)]$. Clearly if
\( \lambda = S(u) < q^* \) and \( \mu = S(l) > q^* \), where \( q^* = S \left( \frac{u+l}{2} \right) \). Hence, to implement the \textit{ex ante} optimal output, the court must induce the firm to experiment. This requires \( S \left( \frac{u+l}{2} \right) \in [S(u), S(2l)] \). Clearly \( S(u) < S \left( \frac{u+l}{2} \right) \) (since \( S \) is a strictly decreasing linear function). \( S \left( \frac{u+l}{2} \right) \leq S(2l) \) implies that \( \frac{u+l}{2} \geq 2l \) and so \( u \geq 3l \).

\[ \square \]

**Proof of Lemma 2.** To implement \( q \in \left[ \frac{\alpha-u}{\beta}, \mu \right] \), the court can simply set \( \lambda = q \) and choose \( F \) large enough, so that the firm is induced to choose \( q_L = \lambda = q \). It is sufficient to choose \( F = \frac{1}{2} \frac{u^2}{\beta} \) - which is the maximum profit the firm can make in the absence of the penalty. With such an \( F \), the firm will never risk a positive probability of receiving the penalty, and hence chooses \( q_L \). To implement \( q = \frac{\alpha}{\beta} \), the court can choose any feasible \( \lambda \in \left[ \frac{\alpha-u}{\beta}, \mu \right] \) and choose \( F \) low enough. It is sufficient to choose \( F = 0 \).

\[ \square \]

**Proof of Proposition 3.** By proposition 2, the court can never induce the firm to choose an output level strictly between \( \mu \) and \( \frac{\alpha}{\beta} \), since if the firm knows it will be fined for sure, it may as well produce at the level that maximizes its pre-penalty profit. Hence if \( q^* > \mu \), it is impossible to implement the efficient output. Noting that \( \frac{\alpha}{\beta} = S(0) \), the second best solution is to choose \( q = \mu \) so long as \( S \left( \frac{u+l}{2} \right) - \mu < S(0) - S \left( \frac{u+l}{2} \right) = \frac{u+l}{2\beta} \). But this condition always holds, since \( \mu \geq S(u) \) (and since \( S \left( \frac{u+l}{2} \right) - \mu \leq S \left( \frac{u+l}{2} \right) - S(u+l) = \frac{u+l}{2\beta} \). By Lemma 2, the court can always entice the firm to choose \( q = \mu \) by simply setting \( \lambda = \mu \) and choosing \( F \geq F_3(\lambda) = \frac{1}{2\beta} T(\mu)^2 \). Alternatively, if \( u \geq 2l \), the court can write a narrower opinion and use the penalty to target \( q = \mu > \lambda \). By proposition 2, the firm will choose \( \mu \) only if \( \hat{\mu}(\lambda) < \mu < S(2l) \). Suppose \( \lambda = S(u) \), then it immediately follows that \( \mu > \hat{q}(\lambda) = \hat{\mu}(\lambda) \) and so the narrow opinion will always suffice. In fact, for any \( \lambda \leq S \left( \sqrt{u^2 - (u - T(\mu))^2} \right) \), \( \mu \geq \hat{q}(\lambda) \), and if so, then choosing \( F \in \left[ F_4(\mu), F_\mu \right] \) will entice the firm to implement \( \mu \), where \( F_4(\mu) = \frac{u-l}{2\beta} \frac{T(\mu)^2}{T(\mu)-l} \) and \( F_\mu = \frac{u-l}{\beta} T(\mu) \). (We need to verify that \( \lambda \leq S(2l(u-l)) \), but this is implied by \( \lambda \leq S \left( \sqrt{u^2 - (u - T(\mu))^2} \right) \) whenever \( u \geq 2l \)).
For $S \left( \sqrt{u^2 - (u - T(\mu))^2} \right) < \lambda < S \left( \sqrt{\frac{u - l}{T(\mu) - l} T(\mu)} \right)$, $\hat{\mu}(\lambda) < \mu < \hat{q}(\lambda)$, and so choosing $F \in [F_4(\mu), F_5(\lambda, \mu)]$ will entice the firm to implement $\mu$, where $F_5 = \frac{u - l}{2\beta} \frac{T(\lambda)^2 - T(\mu)^2}{u - T(\mu)}$.

Suppose instead that $q^* \leq \mu$. Then it is possible to implement the efficient output. Again, by Lemma 2, the court can always entice the firm to choose $q^*$ by simply setting $\lambda = S \left( \frac{u + l}{2} \right)$ and choosing $F$ appropriately. Now, with $\lambda = S \left( \frac{u + l}{2} \right)$, $\lambda \leq S \left( 2\sqrt{l(u - l)} \right)$ requires either $u \leq (7 - \sqrt{32}) l < 2l$ or $u \geq (7 + \sqrt{32}) l$. Hence, unless $u \geq (7 + \sqrt{32}) l$, it suffices to choose $F \geq F_3(\lambda) = \frac{1}{2\beta} (\frac{u + l}{2})^2$. In any case, it is always sufficient to choose $F \geq \frac{u^2}{2\beta}$. Alternatively, if $u \geq 3l$, the court can write a narrower opinion and use the penalty to target $q_M(F) = q^*$. Clearly, since $q_M(F) = S \left( \frac{\beta}{u - l} F \right) = S \left( \frac{u + l}{2} \right)$, the court must choose $F = \frac{u^2 - l^2}{2\beta}$. (To see why beliefs are restricted to $u \geq 3l$, note that, by proposition 2, choosing $q_M$ requires $q_M = S \left( \frac{u + l}{2} \right) \leq S(2l)$.) Choosing $\lambda \leq S \left( \sqrt{u^2 - (u - T(\mu))^2} \right)$ guarantees that $\hat{q}(\lambda) \leq \mu$.

**Proof of Proposition 4.** If $u < 3l$ or $\lambda > S \left( \sqrt{2 \frac{u + l}{2}} \right)$, then the efficient allocation is not implementable. If $u < 2l$ or $\lambda > S \left( 2\sqrt{l(u - l)} \right)$, then this result follows immediately from Proposition 2, since the firm will choose either $\lambda$ or $\frac{u}{\beta}$ (neither of which are efficient, except in the special case of $\lambda = S \left( \frac{u + l}{2} \right)$). Now suppose $u \geq 3l$ and $S \left( \sqrt{2 \frac{u + l}{2}} \right) < \lambda < S \left( 2\sqrt{l(u - l)} \right)$.

Since $\lambda < S \left( \sqrt{2 \frac{u + l}{2}} \right)$ and $u \geq 3l$, then $S \left( \frac{u + l}{2} \right) < \hat{q}(\lambda)$ and so the court cannot target the efficient output using $q_M$. Moreover, $S \left( \sqrt{2 \frac{u + l}{2}} \right) < \lambda$ implies $S \left( \frac{u + l}{2} \right) < \hat{\mu}(\lambda)$ and so court cannot target implement efficiency using $\mu$. (To see this last point, note that $\hat{\mu}(\lambda)$ is defined such that $F_4(\mu) = F_5(\lambda, \mu)$). Setting $\mu = S \left( \frac{u + l}{2} \right)$ implies $F_4 = \frac{1}{\beta} \left( \frac{u + l}{2} \right)^2$ and $F_5 = \frac{1}{\beta} \left[ T(\lambda)^2 - \left( \frac{u + l}{2} \right)^2 \right]$, and so $F_4 \preceq F_5$ whenever $\lambda \leq S \left( \sqrt{2 \frac{u + l}{2}} \right)$.

Now, if $u < 2l$ or $\lambda > S \left( 2\sqrt{l(u - l)} \right)$, then the firm will choose $q \in \left\{ \lambda, \frac{u}{\beta} \right\}$. Since $|\lambda - S \left( \frac{u + l}{2} \right)| < \left| \frac{u}{\beta} - S \left( \frac{u + l}{2} \right) \right|$, implementing $q = \lambda$ is the second best outcome. The court can provide incentives for the firm to choose $\lambda$ by setting $F \geq F_3(\lambda) = \frac{T(\lambda)^2}{2\beta}$. If $2l \leq u < 3l$, the second best policy is for the court to target $q = S(2l)$. By proposition 2, it can do this
by setting $F = \frac{2h(u-l)}{\beta} = F_1$ and $\mu \geq S(2l)$. Finally, if $S\left(\sqrt{2} \frac{u+l}{2}\right) < \lambda < S\left(2\sqrt{I(u-l)}\right)$, then $\hat{\mu}(\lambda) > S\left(\frac{u+l}{2}\right)$. If $|\hat{\mu}(\lambda) - S\left(\frac{u+l}{2}\right)| < |\lambda - S\left(\frac{u+l}{2}\right)|$, then court will set $\mu = \hat{\mu}(\lambda)$ and choose the penalty $F = F_4(\hat{\mu}(\lambda)) = F_5(\lambda, \hat{\mu}(\lambda))$ that implements it. If $|\hat{\mu}(\lambda) - S\left(\frac{u+l}{2}\right)| > |\lambda - S\left(\frac{u+l}{2}\right)|$, the court will seek to implement $q = \lambda$ and it can do this by choosing $F$ large enough.

Suppose $u \geq 3l$ and $\lambda \leq S\left(\sqrt{2} \frac{u+l}{2}\right)$. Then it is possible for the court to implement the efficient outcome. Then by proposition 2, the court can entice the firm to experiment and choose $q_M(F) = S\left(\frac{u+l}{2}\right)$ by setting $F = \frac{u^2-l^2}{2\beta}$ and $\mu \in (S\left(\frac{u+l}{2}\right), S(l))$. (To see this, note that $u \geq 3l$ implies that $S\left(\frac{u+l}{2}\right) \leq S(2l)$ and so $F \geq F_1$. Since $\lambda \leq S\left(\sqrt{u^2 - \left(\frac{u-l}{2}\right)^2}\right)$, then $S\left(\frac{u+l}{2}\right) \geq S\left(u - \sqrt{u^2 - T(\lambda)^2}\right)$ and so $F \leq F_2$. Moreover, since $\mu > S\left(\frac{u+l}{2}\right)$, then $F > F_\mu$ and $\mu > \hat{q}(\lambda)$. Finally, $\lambda \leq S\left(\sqrt{u^2 - \left(\frac{u-l}{2}\right)^2}\right)$ and $u \geq 3l$ implies that $\lambda \leq S\left(2\sqrt{I(u-l)}\right)$. Hence all the conditions for experimentation, as given in proposition 2, are satisfied. ) Alternatively, the court could set $\mu = S\left(\frac{u+l}{2}\right)$ and entice the firm to choose $q = \mu$. This requires $\hat{\mu}(\lambda) \leq S\left(\frac{u+l}{2}\right) \leq S(2l)$, which implies that $u \geq 3l$ and $\lambda \leq S\left(\sqrt{2} \frac{u+l}{2}\right)$.

( If $\lambda \leq S\left(\sqrt{u^2 - \left(\frac{u-l}{2}\right)^2}\right)$ (i.e. if $S\left(\frac{u+l}{2}\right) \geq \hat{q}(\lambda)$), then any $F \in \left[\frac{1}{\beta} \left(\frac{u+l}{2}\right)^2, \frac{u^2-l^2}{2\beta}\right]$ will implement the efficient allocation. If $S\left(\sqrt{u^2 - \left(\frac{u-l}{2}\right)^2}\right) < \lambda \leq S\left(\sqrt{2} \frac{u+l}{2}\right)$, then any $F \in \left[\frac{1}{\beta} \left(\frac{u+l}{2}\right)^2, \frac{1}{\beta} \left(T(\lambda)^2 - \left(\frac{u+l}{2}\right)^2\right)\right]$ will implement the efficient allocation (again by Proposition 2.) It is easily verified that $\lambda \leq S\left(\sqrt{2} \frac{u+l}{2}\right)$ implies $\lambda \leq S\left(2\sqrt{I(u-l)}\right)$.

**Proof of Proposition 5.** First, note that there is no strict incentive to write a broad opinion. Since the lower opinion starts narrow, $\lambda = \hat{\mu}(\lambda) = \hat{q}(\lambda) = S(u)$ - and so, by Lemma 3, any first period outcome that can be generated by a broad opinion can also be generated by a narrow opinion. Moreover, there is no second period benefit to writing a broad opinion, since the court will never choose $\mu_1$ in the second period. (If the experimental level if found to be acceptable, the court will revise up $\lambda$ to perfectly target the socially efficient outcome. If it is found unacceptable, then the court will be able to revise $\mu$ again anyway.)
Next, consider the optimal level of experimentation. If \( u \leq 2l \), then by Proposition 2, the firm will choose either \( q = \lambda = \mathcal{S}(u) \) or \( q = \frac{u}{2} \). Since \( |\mathcal{S}(u) - \mathcal{S}(\frac{u+l}{2})| < \frac{u}{2} - \mathcal{S}(\frac{u+l}{2}) \), the court will implement \( q = \lambda \) by choosing a penalty \( F \geq \frac{u^2}{2} \) and writing any opinion \( \mu \in [\mathcal{S}(u), \mathcal{S}(l)] \).

If \( u > 2l \), the court can induce experimentation in the region \( x \in [\mathcal{S}(u), \mathcal{S}(2l)] \). This output will be found acceptable in the second period with probability \( \frac{T(x)-l}{u-l} \), in which case, the court can revise up its lower opinion \( \lambda_2 \) and induce the socially efficient output in the final period. The second period expected social loss from such a policy (relative to the full information optimum) is:

\[
L^+_2(x) = \frac{1}{2\beta} Var[\theta] + \frac{1}{2\beta} (T(q_2) - E_2[\theta])^2 \\
= \frac{(T(x) - l)^2}{24\beta}
\]

With probability \( \frac{u-T(x)}{u-l} \), \( x \) is found to be unacceptable. The new beliefs are \( (T(x), u) \) and the court will revise down its upper opinion \( \mu_2 \). Since the lower opinion remains narrow, there is again no strict incentive for the court to write a broad opinion in the second period. Recall, by Proposition 4 that the court can only implement the efficient allocation if \( u \geq 3T(x) \). (Note, this requires \( u \geq 6l \) since \( T(x) \geq 2l \).) If \( u < 2T(x) \), the best the court can do is to implement \( q = \lambda = \mathcal{S}(u) \). If \( 2T(x) \leq u < 3T(x) \), the court will implement \( q = \mathcal{S}(2(T(x))) < \mathcal{S}\left(\frac{T(x)+u}{2}\right) \). (Note again, this requires \( u \geq 4l \), since \( T(x) \geq 4l \).) Hence, the expected second period social loss is given by:

\[
L^-_2(x) = \begin{cases} 
\frac{(u-T(x))^2}{6\beta} & x \leq \mathcal{S}\left(\frac{u}{2}\right) \\
\frac{u^2-5uT(x)+7T(x)^2}{6\beta} & \mathcal{S}\left(\frac{u}{2}\right) < x < \mathcal{S}\left(\frac{u}{3}\right) \\
\frac{(u-T(x))^2}{24\beta} & x \geq \mathcal{S}\left(\frac{u}{3}\right)
\end{cases}
\]

In addition, the expected first period loss from choosing \( x \) is: \( L_1(x) = \frac{(u-l)^2}{24\beta} + \frac{(T(x)-\frac{u+l}{2})^2}{2\beta} \).
The court chooses $x$ to minimize the sum of current and future expected losses:

$$
\min_{x \in [S(u), S(2l)]} \mathcal{L} = L_1(x) + \frac{T(x) - l}{u - l} L_2^+(x) + \frac{u - T(x)}{u - l} L_2^-(x)
$$

Suppose the optimal output $x^*$ satisfies $x^* \leq S\left(\frac{u}{2}\right)$. $\mathcal{L}$ is strictly convex in this region, and so the Kuhn-Tucker conditions are sufficient for optimality. The first order condition is:

$$
\frac{\partial \mathcal{L}}{\partial T(x)} = \frac{T(x) - \frac{u + l}{2}}{\beta} + \frac{(T(x) - l)^2}{8\beta(u - l)} - \frac{(u - T(x))^2}{2\beta(u - l)} = 0
$$

which implies that $T(x) = \frac{8 - 2\sqrt{10}}{3}u + \frac{2\sqrt{10} - 5}{3}l$. Note that since $T(x) \geq 2l$, this solution is only feasible if $u \geq \frac{11 - 2\sqrt{10}}{8 - 2\sqrt{10}}l \approx 2.79l$. Hence, for $2l \leq u < 2.79l$, $x^* = S(2l)$. For $u > \frac{11 - 2\sqrt{10}}{8 - 2\sqrt{10}}l$, the optimal net social loss is:

$$
\mathcal{L}(x^*) = \frac{(u - l)^2}{24\beta} \left[ \frac{520 - 160\sqrt{10}}{9} \right].
$$

I now show that the optimal solution must be contained in the region $x \leq S\left(\frac{u}{2}\right)$. Suppose the optimal solution is in the region $x \geq S\left(\frac{u}{3}\right)$. Now, in this region, $\frac{\partial \mathcal{L}}{\partial T(x)} = \frac{(T(x) - \frac{u + l}{2})}{\beta} + \frac{(T(x) - l)^2}{8\beta(u - l)} - \frac{(u - T(x))^2}{8\beta(u - l)} < 0$ since $T(x) \leq \frac{u}{3}$. Hence the optimum is at $T(x) = \frac{u}{3}$. But $\mathcal{L}\left(S\left(\frac{u}{3}\right)\right) = \frac{5(u - l)^2}{72\beta} + \frac{6u - 5l^2}{36\beta} > \mathcal{L}(x^*)$. Hence, the optimal solution cannot exist in region $x \geq S\left(\frac{u}{3}\right)$. A similar argument shows that it cannot be in the region $S\left(\frac{u}{2}\right) < x < S\left(\frac{u}{3}\right)$ either. 

**Proof of Proposition 6.** First, using the same argument as in the proof of Proposition 5, the court will optimally experiment in the region $x \leq S\left(\frac{u}{2}\right)$. However, unlike the previous case, there is now a strict incentive for the court to write a broad opinion $\lambda$. To see why, if, in the second period, the experimental level $x^*$ is found to be unacceptably high, then by Proposition 4, the best the second period court can do is to implement $\lambda$. Writing a broad opinion can bring this second period choice closer to the efficient second period level, than writing a narrow opinion. (Of course, the opinion cannot be so broad as to prevent experimentation.)
The court’s problem is:

$$\min_{x, \lambda} \mathcal{L} = \frac{(u - l)^2}{24\beta} + \frac{(T(x) - \frac{u + l}{2})^2}{2\beta} + \frac{(T(x) - l)^3}{24\beta(u - l)} + \frac{u - T(x)}{u - l} \left[ \frac{(u - T(x))^2}{24\beta} + \frac{(T(\lambda) - \frac{u + T(x)}{2})^2}{2\beta} \right]$$

s.t $$T(x) \leq u - \sqrt{u^2 - T(\lambda)^2}$$

Letting $$\phi$$ denote the Lagrange multiple, the first order conditions are:

$$\frac{\partial L}{\partial T(x)} = \frac{T(x) - \frac{u + l}{2}}{\beta} + \frac{(T(x) - l)^2}{8\beta(u - l)} - \frac{(u - T(x))^2}{8\beta(u - l)} - \frac{(T(\lambda) - \frac{u + T(x)}{2})^2}{2\beta(u - l)} - \frac{u - T(x) T(\lambda) - \frac{u + T(x)}{2}}{u - l}$$

$$\frac{\partial L}{\partial T(\lambda)} = \frac{u - T(x)}{u - l} \frac{T(\lambda) - \frac{u + T(x)}{2}}{\beta} + \phi \frac{T(\lambda)}{\sqrt{u^2 - T(\lambda)^2}} = 0$$

Hence, if the constraint does not bind, (3) implies that $$T(\lambda) = \frac{u + T(x)}{2}$$, and so (2) implies that $$T(x) = \frac{u + l}{2}$$. But this solution does not satisfy the constraint. (It is easy to verify that $$\frac{u + l}{2} \leq u - \sqrt{u^2 - \left(\frac{3u + l}{4}\right)^2}$$ only if $$-\frac{5}{3}l \leq u \leq l$$.) Hence, the constraint binds and so $$T(x) = u - \sqrt{u^2 - T(\lambda)^2}$$. (Alternatively, $$T(\lambda) = \sqrt{u^2 - (u - T(x))^2}$$.)

The court’s problem now becomes:

$$\max_x \mathcal{L} = \frac{(u - l)^2}{24\beta} + \frac{(T(x) - \frac{u + l}{2})^2}{2\beta} + \frac{(T(x) - l)^3}{24\beta(u - l)} + \frac{u - T(x)}{u - l} \left[ \frac{(u - T(x))^2}{24\beta} + \frac{\sqrt{u^2 - (u - T(x))^2} - \frac{u + T(x)}{2}}{2\beta} \right]$$

and the first derivative is:

$$\frac{\partial \mathcal{L}}{\partial T(x)} = \frac{T(x) - \frac{u + l}{2}}{\beta} + \frac{(T(x) - l)^2}{8\beta(u - l)} - \frac{(u - T(x))^2}{8\beta(u - l)} - \frac{(T(\lambda) - \frac{u + T(x)}{2})^2}{2\beta(u - l)} + \frac{u - T(x) T(\lambda) - \frac{u + T(x)}{2}}{u - l} \frac{1}{T(\lambda)}$$

(4)

If $$T(x) = u$$, then $$T(\lambda) = u$$ and so: $$\frac{\partial \mathcal{L}}{\partial T(x)} = \frac{5(u - l)}{8\beta} > 0$$. Hence $$T(x^*) < u$$ and so $$x^* > S(u)$$. By contrast, $$x^* < S\left(\frac{u + l}{2}\right)$$, by the same argument as in the proof of Proposition 5. (Since
$T(\lambda) > \frac{3u+l}{4}$, the loss when $x = \frac{u+l}{2}$ is discovered to be too high is larger than the loss when it is found to be acceptable. But these each occur with equal probability.) Hence $S(u) < x^* < S\left(\frac{u+l}{2}\right)$.

**Proof of Proposition 7.** Suppose the firm chooses $q$ in the first period. With probability $\frac{T(q)-t}{u-l}$, this policy is found to be acceptable. The court will write a broad opinion at the *ex ante* optimal level $\lambda_2 = S\left(\frac{T(q)+t}{2}\right)$ and set a penalty large enough so that the firm chooses this in the final period. By contrast, with probability $\frac{u-T(q)}{u-l}$, the policy is found to be unacceptable, and the court will implement $q = \lambda_1$ in the following period. Noting that $\alpha q - \frac{1}{2} \beta q^2 = \frac{a^2-T(q)^2}{2\beta}$, the firm’s continuation payoff is:

$$\Pi(q; \lambda_1, \mu_1, l_1, u_1) = \frac{u_1-T(q)}{u_1-l_1} \frac{\alpha^2-T(\lambda_1)^2}{2\beta} + \frac{T(q)-l_1}{u_1-l_1} \left[\frac{\alpha^2-T(q)+l_1}{2\beta}\right]$$

Now, $\frac{\partial \Pi}{\partial q} = -\frac{T(\lambda_1)^2}{2(u-l)} - \frac{T(q)}{2(u-l)} = -\frac{T(\lambda_1)^2+T(q)}{2(u-l)} < 0$. If the firm experiments, then it will choose $q$ to maximize its stream of current and future payoffs. Hence, it maximizes: $\alpha q - \frac{1}{2} \beta q^2 - \frac{u-T(q)}{u-l} F + \Pi(q)$. The first order condition is:

$$\alpha - \beta q - \frac{\beta F}{u-l} + \frac{\partial \Pi}{\partial q} = 0$$

Recall that the largest experimental level that the firm could be induced to choose was $q = S(2l)$, and it would choose this when $F = F_1 = 2l \frac{u-l}{\beta}$. This output had the property that $\alpha - \beta S(2l) - \frac{\beta F_1}{u-l} = 0$. But the clearly, $\alpha - \beta S(2l) - \frac{\beta F_1}{u-l} + \frac{\partial \Pi}{\partial q} < 0$. 

\[ \square \]
References


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