Delays and Partial Agreements in Multi-Issue Bargaining

Avidit Acharya†        Juan Matias Ortner‡

July 2010

Abstract

We model a situation in which two players bargain over two issues (pies), one of which can only be resolved at a future date. We find that if the players value the issues asymmetrically (one player considers the existing issue more important than the future one, while the other player has the opposite valuation) then they may delay agreement on the first issue until the second one is finally on the table. If we allow for partial agreements, then the players never leave an issue completely unresolved. They either make a partial agreement on the first issue, and wait for the second one to emerge before completing the agreement; or they come to complete agreements on each of the issues at their earliest possible dates. We discuss applications to legislative bargaining and international trade negotiations.

JEL Classification Codes: C73, C78
Key words: bargaining, multiple issues, delay, hold-up, inefficiency

*We are grateful to Christina Davis, John Londregan, Adam Meirowtiz, Stephen Morris and Kris Ramsay for valuable discussions and comments. We are especially indebted to Faruk Gul for his encouragement and guidance.

†Woodrow Wilson School of Public and International Affairs, Robertson Hall, Princeton University, Princeton NJ 08544-1013 (e-mail: aacharya@princeton.edu).

‡Department of Economics, Fisher Hall, Princeton University, Princeton NJ 08544-1021 (e-mail: jortner@princeton.edu).
1 Introduction

The noncooperative theory of bargaining, introduced by Rubinstein (1982), deals with situations in which two players seek an agreement on how to divide a surplus. Once an agreement is reached, each player receives his share and the game ends with the two players never interacting again in the future. However, as Schelling (1960) noted many years ago, several real life situations are ones in which the bargaining parties negotiate over not one but multiple issues. While bargaining over an issue today, the players may, for instance, know that in the future they will come to the negotiating table again to bargain over a new issue altogether.

In this paper, we build a model to capture such a situation. We assume that there are two pies, one of which is already on the table.\(^1\) The other pie can only be consumed starting at a random future date.\(^2\) The players have opposite valuations for the two pies: one player values the first pie more than the second, while the other player values the second more than the first.\(^3\) The players can either bargain over the two pies as they arrive, or wait for the second pie to arrive so that they bargain over the two pies simultaneously. We first consider a model in which the players are constrained to make complete offers. In other words, each pie must either be completely consumed or remain completely unconsumed. We show that in some circumstances (i.e. for some parameter values) the players may wait for the second pie to arrive before coming to an agreement on the first.

Although our setup is highly stylized, there are many real life examples that have its structure. Consider the following example. An economics department has two openings—one for a political economist and one open search—that it would like fill over the next couple years. Suppose that a candidate appears that is attractive to both the political economy faculty and those who specialize in theory, but more so to the political economists. To hire her, the political economists need the support of the theorists, and would like to extend her an offer under the *quid pro quo* understanding that when an excellent theorist who does some political economy arrives on the market, they

---

\(^1\)We use the terms “pie,” “issue” and “surplus” interchangeably.

\(^2\)The assumption that the second pie is available only in the future may be interpreted as a physical constraint on the bargaining environment or may capture the idea that some issues are not yet ripe for discourse.

\(^3\)Our model, therefore, differs in many ways from the one considered by Muthoo (1995), who develops a model of repeated bargaining where new issues emerge only after agreements have been reached on existing ones; and in which all issues are strategically identical, i.e. there is no asymmetry in valuations as we have assumed here.
will support the theorists in their bid to make that candidate an offer. But they cannot commit to supporting the theorists in any of their future bids. If the department makes an early offer to the existing candidate, then the political economy faculty can afford to be obstinate in the future and seek another of their own for the open search. Because of the commitment problem, the theorists may want to wait and see if the candidate that they are looking for appears on the market in the same year. If she does, then the department could make the two offers simultaneously. But if they wait too long, the department suffers the risk that the existing candidate takes a job at a competing institution. In other words, delay is costly.

The example provides some insight into the role of commitment problems in producing delays in bargaining situations. If the players in our model come to an early agreement on the first pie (and immediately consume the benefits of this agreement), then the subgame that starts when the second pie arrives is a standard bargaining game a la Rubinstein. Consequently, the player that values the second pie less will still demand a large share of the surplus if the offer that is on the table is to be accepted. In other words, the agreement will be such that both parties receive a sizable portion of the surplus regardless of how they value it. On the other hand, if the players delay an agreement on the existing pie until the second one arrives, then each player will receive a larger share of the pie that he values more. Therefore, if discounting between periods is small, and if the expected waiting time for the second issue is not too long, then the players prefer to wait and bargain over the two pies simultaneously.

After analyzing the case of complete offers, we consider a model that is otherwise the same, except that players are able to make partial offers, consuming only a fraction of the existing surplus and saving the remainder for future negotiations. In this case, the players may come to a partial agreement on the first pie, completing the agreement only when the second one arrives. In other words, for some parameter values, they consume some (but not all) of the pie in the first period, and the remainder in the period that the second pie arrives. Therefore, even the partial offers case features a form of delay; but, more importantly, it provides a framework for understanding situations in which we may observe partial agreements—something quite new to the bargaining literature.

The intuition for the partial offers case is as follows. Since the players are able to save fractions of the existing pie for future consumption, they are better able to balance the tradeoff between the benefits of an early agreement and the efficiency gains from bargaining over the two pies together. Indeed, even when discounting is small and the expected waiting time for the second pie is short, the players will agree to consume a
positive fraction of the first pie in the first round of negotiations. They then complete the agreement when the second pie arrives.

We stress that if the players could commit to an agreement on the second issue before it is on the table (for instance, if they could write enforceable contracts), then they will always be able to implement a Pareto efficient allocation. However, in the absence of a commitment mechanism, any early agreement that the players may reach on the second issue will be violated with certainty once it is finally on the table. Moreover, any equilibrium outcome of our game is inefficient, either because players delay, or because immediate agreements on each of the issues involve inefficient allocations of the pies.\footnote{If the players immediately agree, then they both consume a positive fraction of each pie while efficiency requires that at least one player consumes all of the pie he values more.}

The inefficiencies arise naturally as a result of the timing of the game (the fact that the second pie is not yet on the negotiating table) and the assumption that parties attach different values to each of the pies. In particular, if the two parties both value the two pies the same then they come to complete and immediate agreements on both pies. But if there is an asymmetry in valuations—and this asymmetry is large—then the players may delay making a complete agreement on the first issue until the second one arrives.

\subsection{1.1 Related Literature}

Our paper is related primarily to the literature on bargaining impasses. Kennan and Wilson (1993) provide a review of bargaining models with incomplete information involving delay. Babcock and Loewenstein (1997) provide empirical evidence about differences in beliefs among parties in bargaining situations, and the role these divergent beliefs might have in explaining bargaining delays. Yildiz (2004) offers a framework that explains delay as the outcome of each of the players having overly optimistic beliefs about their future bargaining power. Abreu and Gul (2000) show that if each player believes that the other player might be irrational, then delay can arise in equilibrium since by delaying an agreement a player can build a reputation for being irrational. Compte and Jehiel (2004) introduce a bargaining model with complete information and history-dependent outside options to show that parties will make only gradual concessions until a final agreement is reached. Finally, Merlo and Wilson (1998) consider a model in which players delay agreement as they wait for the pie to increase in size. Delay in this model is, however, an efficient outcome.\footnote{An additional difference between the Merlo-Wilson setup and ours is that in their model the game ends as soon as there is agreement; however, in our model because there are two pies, the game continues}
Because of the presence of the second issue, our paper is also closely related to the literature on agenda bargaining. In an early paper, Fershtman (1990) showed that in multi-issue bargaining, the agenda (the order in which the issues are negotiated) might have an influence on the outcome of the game; and that if players have conflicting valuations over the issues at stake, then the outcome might be inefficient. His focus, however, is on disagreements over the agenda, while our paper takes the agenda as fixed. Inderst (1998) studies a multi-issue bargaining model in which players can freely choose the subset of issues over which they make offers. He shows that if all issues are mutually beneficial (they yield positive utility to both parties) then players come to an immediate agreement on all projects, while if there are some strictly controversial issues then the game has multiple equilibria, some of which involve delay. In and Serrano (2003, 2004) also study bargaining models with an endogenous agenda. They show that if players can only make offers about one issue at a time, and frictions are small, then the game has multiple equilibria. However, if players can make offers on any of the remaining issues, then under some conditions the game has a unique outcome.

Finally, our paper is related to the literature on the hold-up problem. This literature models situations in which one player does not take an efficient action (e.g. an investment) because it would increase the bargaining power of the other player (Grout 1984; Grossman and Hart 1986).\(^6\) Similarly, in our paper, the player who values the future pie can immediately agree on a division of the first pie but may prefer to delay, since agreeing would increase the bargaining power of his opponent when bargaining over the second pie.

2 The Model

There are two players \(i = 1, 2\) and two pies \(X\) and \(Y\). Time is discrete and indexed by \(t = 0, 1, 2, \ldots\). If \(x_{it}\) and \(y_{it}\) are the shares of pies \(X\) and \(Y\) consumed by player \(i\) in period \(t\), then the players’ stage game utilities are given by

\[
\begin{align*}
    u_{1t}(x_{1t}, y_{1t}) &= x_{1t} + r_1 y_{1t} \\
    u_{2t}(x_{2t}, y_{2t}) &= r_2 x_{2t} + y_{2t}
\end{align*}
\]

\(^6\)For proposed solutions to the hold-up problem, see Rogerson (1992) and Gul (2001).
where $0 < r_1 \leq 1$ is player 1’s marginal rate of substitution between pies and $1/r_2 \geq 1$ is the corresponding quantity for player 2. Players’ preferences over consumption sequences are represented by $\sum_{t \geq 0} \delta^t u_{it}(x_{it}, y_{it})$, where $\delta$ is a common discount factor.

In every period, each of the two players is recognized with probability $1/2$ to be proposer. The proposer’s task is to make an offer of nonnegative consumption shares $\{x_{1t}, x_{2t}, y_{1t}, y_{2t}\}$. The other player, whom we call the responder, must then either accept or reject the proposal. If the offer is accepted, then the proposed shares are consumed and the period ends; if the offer is rejected, then each player consumes a share $V$ of each pie and the period ends.

The only restriction on proposals is that they be feasible, and feasibility is state-contingent. In each period the state is given by $(k_t, s_t)$ where $k_t = 1, 2$ is the identity of the proposer and $s_t$ determines the fractions of pies $X$ and $Y$ that can be consumed in that period.\footnote{We sometimes abuse terminology and refer only to component $s_t$ as the state.} We assume that pie $X$ exists beginning in period 0, and thus part or all of it may be consumed starting in the first period. On the other hand, pie $Y$ arrives stochastically: if it has not arrived by period $t$, then it arrives at the beginning of period $t + 1$ with probability $p < 1$. Obviously, no fraction of the pie can be consumed before it has arrived.

As is standard, our equilibrium concept throughout the paper is subgame perfect equilibrium, which we refer to simply as equilibrium. Let $\lambda_t = 1 - \sum_{t' < t} (x_{1t'} + x_{2t'})$ be the share of pie $X$ not yet consumed by period $t$, with the convention $\lambda_0 = 1$. In any subgame in which pie $Y$ has already arrived, the players will, in any equilibrium, come to an agreement over all of pie $Y$ and all of the fraction of pie $X$ that has not yet been consumed (and thus the game will end). In other words, if pie $Y$ arrives in period $t$ then the players consume the total shares $x_{1t} + x_{2t} = \lambda_t$ and $y_{1t} + y_{2t} = 1$. This implies that there is no loss of generality in writing the state $s_t$ as either $\lambda_t X$ if pie $Y$ has not yet arrived and $\lambda_t XY$ if it has. Feasibility then requires that proposals satisfy

$$x_{1t} + x_{2t} \leq \lambda_t \quad \text{and} \quad y_{1t} + y_{2t} \leq \psi_t = \begin{cases} 0 & \text{if } s_t = \lambda_t X \\ 1 & \text{if } s_t = \lambda_t XY. \end{cases}$$

Suppose the players consume a total $x_{1t} + x_{2t} = \mu_t$ of pie $X$ in period $t$. Our assumptions on the timing of the game imply that if the state in period $t$ is $\lambda_t X$, then the state in the next period is $\lambda_{t+1} XY$ with probability $p$ and $\lambda_{t+1} X$ with probability $1 - p$, where in both cases $\lambda_{t+1} = \lambda_t - \mu_t$. As mentioned before, in equilibrium the states
\( \lambda_t XY \) are terminal; that is, the players consume \( x_{1t} + x_{2t} = \lambda_t \) and \( y_{1t} + y_{2t} = 1 \) when the state is \( \lambda_t XY \). Finally, for notational convenience, if \( \lambda_t = 0 \) we write the states as \( \emptyset = \lambda_t X \) and \( Y = \lambda_t XY \), while if \( \lambda_t = 1 \) we write \( X = \lambda_t X \) and \( XY = \lambda_t XY \).

We say that the players reach a complete agreement at state \((j, s_t)\) if they consume totals \( x_{1t} + x_{2t} = \lambda_t \) and \( y_{1t} + y_{2t} = \psi_t \). We say that they delay at state \((j, s_t)\) for \( s_t \neq \emptyset \) if they consume \( x_{1t} + x_{2t} = 0 \) and \( y_{1t} + y_{2t} = 0 \). If the state is \( s_t = \emptyset \), we say that the players are waiting. Finally, we say that the players reach a partial agreement at state \((j, s_t)\) in all other cases, i.e. if they do not delay, wait or completely agree.

In the next section, we characterize the equilibrium of the game in which proposals must satisfy (1) with both inequality signs replaced by equalities. This is analytically equivalent to assuming that fractions of pie \( X \) cannot be stored. Section 4 then characterizes the equilibrium of the game in which the constraints in (1) are satisfied, as stated. In contrast with existing models, we show that the equilibrium may involve partial agreement on \( X \). Finally, in Section 5 we show that the main qualitative insights of our two models are robust to the removal of bargaining frictions. We also report the shares of each pie consumed by the players in the continuous time limit.

### 3 Complete Agreements

In this section, players’ proposals are constrained to be complete agreements; that is, the feasibility constraints are given by (1) with the two inequalities replaced by equalities. This assumption further simplifies the states: we have \( s_t \in \{X, \emptyset, Y, XY\} \) for all \( t \). Each of these states describes one of the four possibilities: (i) only pie \( X \) exists so far, but no agreement on its division has been made, which we call state \( X \); (ii) \( X \) has already been consumed, but \( Y \) has not yet arrived, which is the waiting state, \( \emptyset \); (iii) \( X \) has been consumed, and \( Y \) has emerged but the players have yet to agree on \( Y \), which we call state \( Y \); and (iv) \( Y \) has arrived and neither \( X \) nor \( Y \) have been consumed, which we call \( XY \).

In states \( Y \) and \( XY \), a Rubinstein-style (1982) argument establishes that equilibrium payoffs are unique for each \((j, Y)\) and \((j, XY)\), and that these states are terminal. Moreover, the equilibrium profile prescribes an essentially unique behavior rule for each player in each of these states. Working backwards, this implies that equilibrium payoffs in states \((j, \emptyset)\) are also unique. Therefore, the main step in establishing the uniqueness of equilibrium payoffs is to show that the equilibrium payoffs in states \((j, X)\) are unique as well.
Theorem 1 There are unique equilibrium payoffs that can be supported by a profile of stationary strategies. In any equilibrium the players reach complete agreements in all subgames beginning in states $Y$ and $XY$. For any subgame beginning in state $X$, there exists a threshold $\bar{p}(\delta, r_1, r_2)$ such that in any equilibrium the players reach a complete agreement when $p < \bar{p}(\delta, r_1, r_2)$ and delay when $p > \bar{p}(\delta, r_1, r_2)$.

In Appendix 8.1 we show that the equilibrium payoffs are unique and can be supported by a profile of stationary strategies. Thus we can define $v_i(j, s)$ to be the equilibrium payoff for player $i$ in state $(j, s)$. It will be useful to also define

$$w_i(\lambda X) = \frac{1}{2} v_i(1, \lambda X) + \frac{1}{2} v_i(2, \lambda X)$$

$$w_i(\lambda XY) = \frac{1}{2} v_i(1, \lambda XY) + \frac{1}{2} v_i(2, \lambda XY)$$

$$W(s) = r_2 w_1(s) + w_2(s)$$

where $s = \lambda X$ or $\lambda XY$. The first two quantities are the ex ante expected values for states $\lambda X$ and $\lambda XY$ respectively. The third quantity, $W(s)$, can be thought of as the expected total surplus for states $s$, though this interpretation is not quite accurate even for these states because the game does not admit transferable utility. We nevertheless refer to $W(s)$ as the total payoff for state $s$. The remaining proof of Theorem 1 is as follows.

Proof. (Result for State X Only) If there is complete agreement in state $X$, the proposal $\{x_1, x_2\}$ must satisfy both

$$x_1 + w_1(\emptyset) \geq \delta [pw_1(XY) + (1 - p)w_1(X)]$$

$$r_2 x_2 + w_2(\emptyset) \geq \delta [pw_2(XY) + (1 - p)w_2(X)]$$

The first inequality states that what player 1 gets if the proposal is accepted should not be less than his continuation value from making a proposal that player 2 will not accept, if he is the proposer, or rejecting the offer if he is the responder. The second inequality states the same for player 2. The feasibility constraint $0 \leq x_1 \leq 1$ is implied by these inequalities. We substitute $x_1 = 1 - x_2$ in the inequalities, solve for $x_2$ in each, and combine to get:

$$r_2 + w_2(\emptyset) - \delta[pw_2(XY) + (1 - p)w_2(X)]$$

$$\geq r_2 \delta[pw_1(XY) + (1 - p)w_1(X)] - r_2 w_1(\emptyset).$$

(3)
We note the fact that $W(\emptyset) = \alpha W(Y)$, where $\alpha = \delta p/(1 - \delta(1 - p))$. This is an immediate consequence of (A1) in the Appendix. Further, complete agreement at state $X$ implies

$$W(X) = r_2 w_1(X) + w_2(X) = r_2 + r_2 w_1(\emptyset) + w_2(\emptyset) = r_2 + W(\emptyset).$$

Applying these conclusions and other definitions, we find that (3) reduces to

$$r_2 + \alpha W(Y) \geq \alpha W(XY). \quad (4)$$

The quantities $\alpha$, $W(Y)$ and $W(XY)$ depend only on the parameters $p$, $\delta$, $r_1$ and $r_2$ (see Appendix 8.2 for the calculations), so that (4) may be solved for $p$ to get

$$p \leq \bar{\rho}(\delta, r_1, r_2) = \frac{2r_2(1 - \delta)(4(1 - \delta) + \delta^2(1 - r_1 r_2))}{\delta^2(1 - r_1 r_2)[2(1 + r_2(1 - \delta)) - \delta(1 + r_1 r_2)]}. \quad (5)$$

Conversely, if (5) holds with strict inequality then the uniqueness of equilibrium payoffs implies that the players will reach a complete agreement in state $X$. If (5) does not hold, then complete agreement at state $X$ cannot be part of the equilibrium outcome. Hence there must be delay. ■

Theorem 1 shows that whether or not there is agreement in state $X$ does not depend on the identity of the proposer: the only quantities that matter are the total payoffs $W(Y)$ and $W(XY)$. Moreover, the theorem highlights the tradeoff between the benefits of an immediate agreement on pie $X$ and the efficiency gains from bargaining over $X$ and $Y$ together. If the expected waiting time for pie $Y$ to arrive is short (i.e. $p > \bar{\rho}(\delta, r_1, r_2)$) then the efficiency gains from bargaining the two pies simultaneously outweigh the benefits from not delaying; and hence the players delay an agreement on $X$ until pie $Y$ arrives. Indeed, inequality (4) states that the total payoff from agreeing should exceed the total discounted payoff from delaying on $X$ until $Y$ arrives. If the two sides of (4) hold with equality, then the only offer that the proposer can make that supports agreement is one that makes him indifferent between agreeing on $X$ now and waiting for pie $Y$. (This is a knife-edge case that supports both agreement and delay as equilibrium outcomes.) Clearly, agreement is the unique outcome when the left side of this inequality strictly exceeds the right.
The threshold $\tilde{p}(\delta, r_1, r_2)$ is strictly decreasing in $\delta$ and strictly increasing in $r_1$ and $r_2$. Therefore, the comparative static findings are that the delay region shrinks as $\delta$ or $p$ decrease and $r_1$ and $r_2$ increase. As the players become more patient, it is more likely that they will delay consuming $X$ until pie $Y$ arrives, and if pie $Y$ is expected to arrive soon they are even more likely to delay. We also highlight that asymmetric valuations (the assumption that $r_1, r_2 < 1$) is crucial for generating delay. If $r_1 = r_2 = 1$, i.e. the players each value the two pies equally, then they never delay in state $X$ because there are no efficiency gains to be made from doing so.

**Example.** To illustrate the situation where the players delay making an agreement on $X$ until the arrival of $Y$, we focus on the special case of $r_1 = r_2 = r < 1$, which we call the *symmetric case*. This assumption reduces the dimensionality of the parameter space, so that whether or not delay takes place in state $X$ depends only on $\delta$, $p$ and $r$. Figure 1 plots the threshold $\tilde{p}$ viewed as a function of $r$ for fixed values of $\delta$. For each value of $\delta$, the complete agreement region is the part of the box strictly to the right of the plotted curve, while the delay region is strictly to the left.\(^8\)

\(^8\)Inspecting the figure allows us to guess parameter values for which delay will take place in state $X$. Consider, for example, $p = r = 0.5$ and $\delta = 0.9$. Then, the right side of (4) is 0.653 while the left side is 0.556. Therefore, there is no equilibrium such that there is agreement in state $X$. Hence, there must be delay.
4 Partial Agreements

We now return to the assumption that all proposals must satisfy the feasibility constraints in (1), as stated. That is, in this section we consider the bargaining game in which players can make partial offers over existing pies and leave the remainder for future negotiations. In this setup the possible states in any period $t$ are $\lambda X$ and $\lambda XY$ with $0 \leq \lambda \leq 1$. The main result of this section is the following.

**Theorem 2** There are unique equilibrium payoffs that can be supported by a profile of stationary strategies. In any equilibrium, the players reach a complete agreement in all subgames beginning in state $\lambda XY$. For any subgame beginning in state $\lambda X$ there exist thresholds $\hat{p}(\delta, r_1, r_2)$ and $\hat{\lambda}(\delta, r_1)$ such that in any equilibrium:

(i) if $p < \hat{p}(\delta, r_1, r_2)$, the players reach a complete agreement, and

(ii) if $p > \hat{p}(\delta, r_1, r_2)$, the players reach a partial agreement (consuming $x_1 + x_2 = \lambda - \hat{\lambda}(\delta, r_1)$) when $\lambda > \hat{\lambda}(\delta, r_1)$, and they delay when $\lambda \leq \hat{\lambda}(\delta, r_1)$.

Unlike the case of complete agreements, when partial offers are allowed, the essentially unique equilibrium always involves some form of agreement (either complete or partial) in the initial round of negotiations. To understand the intuition behind this, note that the main tradeoff that the players face when only pie $X$ is on the table is between the benefits of an early agreement and the efficiency gains from bargaining over the two pies together (and these gains can only be realized if agreement is delayed until $Y$ arrives). The key point is that allowing for partial offers gives the players additional flexibility to balance this tradeoff in a smooth fashion. Indeed, in this setup when complete agreement on $X$ is not possible, the players will choose to consume a positive fraction of this pie at the beginning of the game, saving the remainder until the arrival of pie $Y$.

The theorem states that any state $\lambda XY$ is a terminal state. For states $\lambda X$, the threshold $\hat{p}(\delta, r_1, r_2)$ is comparable to the threshold $\tilde{p}(\delta, r_1, r_2)$ in the case of complete offers. The theorem also shows that the players will reach a partial agreement on pie $X$ at the beginning of the game if $p > \tilde{p}(\delta, r_1, r_2)$, and will reach a complete agreement if $p < \tilde{p}(\delta, r_1, r_2)$. Similarly, when offers are restricted to be complete, Theorem 1 shows that the players will delay an agreement on pie $X$ if $p > \tilde{p}(\delta, r_1, r_2)$, and they will reach a complete agreement if $p < \tilde{p}(\delta, r_1, r_2)$. One can show that

$$\tilde{p}(\delta, r_1, r_2) > \hat{p}(\delta, r_1, r_2)$$
for all $\delta, r_1, r_2 < 1$. In other words, there is a region of the parameter space in which the players would come to a complete agreement on $X$ if constrained to make complete offers, but would arrive at a partial agreement if they had the option. Equivalently, if the players were to reach a complete agreement in the model with partial offers, they would reach a complete agreement in the model with complete offers as well; however, the converse is not true. In this sense, the added flexibility from partial offers may in fact hinder the chances of reaching a complete agreement in the early stages of the game.

To understand how the threshold $\hat{\lambda}(\delta, r_1)$ is determined, consider a terminal state $\lambda_{XY}$. At any such state the proposer will make an offer that will leave the responder indifferent between accepting and rejecting. Suppose that $\lambda$ is very high; for example, it is close to 1. If player 2 is the proposer, then to make player 1 indifferent between accepting and rejecting, player 2 can optimally propose all of pie $Y$ to himself, as well as a fraction of pie $X$ as proposer’s rent. If $\lambda$ is very low, then to make player 1 indifferent between accepting and rejecting, player 2 must not only offer all $\lambda$ of pie $X$ to player 1, he must also offer some of pie $Y$ – the pie he values more. The threshold $\hat{\lambda}(\delta, r_1)$ that we calculate in the appendix is the point at which player 2 must begin to offer some of pie $Y$ to player 1.

At the threshold $\hat{\lambda}(\delta, r_1)$ there is a change in the tradeoff between the benefits of immediately consuming an additional fraction of pie $X$ and the efficiency gains from saving that fraction until pie $Y$ arrives. In particular, when $\lambda$ reaches $\hat{\lambda}(\delta, r_1)$ the foregone gains of saving $X$ increase because of the change in the way pies are allocated in equilibrium when $Y$ arrives. This is why for some parameter values, the players prefer to stop consuming pie $X$ when they have consumed $1 - \hat{\lambda}(\delta, r_1)$ of it.

5 Frequent Offers

In this section, we study properties of equilibrium without bargaining frictions. We show that the qualitative insights of our previous analysis continue to hold even as bargaining frictions vanish. In particular, we prove that the inefficiency arising from the delayed consumption of pie $X$ persists even as players make offers arbitrarily frequently. We illustrate this in the symmetric case with $r_1 = r_2 = r$, although a similar analysis would hold if we allowed for arbitrary $r_1 \neq r_2$.

Let $T = \{0, \Delta, 2\Delta, \ldots\}$ be the set of dates at which offers are made. In other words, $\Delta$ is the time interval between any two consecutive bargaining rounds. We will denote by $\delta(\Delta) = e^{-\rho\Delta}$ the discount factor, where $\rho$ measures the time preferences of the agents.
Finally, we assume that if pie $Y$ has not arrived by date $t\Delta$, then it arrives at date $(t + 1)\Delta$ with probability $p(\Delta) = 1 - e^{-\varphi\Delta}$, where $\varphi$ is the arrival rate. The game is therefore fully characterized by the parameters $(r, \Delta, \varphi, \rho)$.

We first analyze the case of complete offers (as in Section 3). In this case, as $\Delta \to 0$ there exists $\varphi(r, \rho) > 0$ such that in equilibrium players come to an immediate agreement in state $X$ if and only if $\varphi \leq \varphi(r, \rho)$. To see this, note that our analysis from Section 3 shows that for any $\Delta > 0$ there is agreement in state $X$ only if

$$p(\Delta) \leq \frac{2r}{\delta(\Delta)(1 + r)(1 - r^2)} \frac{2 - \delta(\Delta)(1 - r)}{\delta(\Delta)}.$$  

Substituting $\delta(\Delta) = e^{-\rho\Delta}$ and $p(\Delta) = 1 - e^{-\varphi\Delta}$ and taking limits as $\Delta \to 0$, this inequality reduces to

$$\varphi \leq \varphi(r, \rho) = \frac{2r \rho}{1 - r^2}.$$  

Therefore, just as in the discrete case, in the continuous time limit agents reach an immediate agreement in state $X$ only if the rate at which $Y$ arrives is small relative to the discount rate $\rho$ and the efficiency gains from bargaining over the two issues together (which depend on $r$).

We get simple closed form expressions for the fractions of each of the pies that the players consume in the limit as $\Delta \to 0$. Denote the period in which pie $Y$ arrives $t_Y$. When agreement on $X$ is delayed until $t_Y$ (i.e. when $\varphi > \varphi(r, \rho)$), then in the continuous time limit each player consumes the entirety of the pie he likes more, as indicated in Table 1. This follows from equation (A9) in Appendix 8.2 (setting $\lambda = 1$) by taking the limit as $\delta \to 1$.

On the other hand, if $\varphi \leq \varphi(r, \rho)$ the players reach an immediate agreement on pie $X$ and in the limit as $\Delta \to 0$ the consumption shares of pie $X$ are given by the corresponding expressions in Table 1. Since $r < 1$, one can easily check from the expressions in the table that $x_1^1 = x_1^2 < 1/2$ and $x_2^1 = x_2^2 > 1/2$, so that player 2 consumes a larger fraction of pie $X$ even though he values it less than player 1. The intuition behind this is that the cost of delay is larger for player 1 than for player 2 when pie $Y$ has not yet arrived, since a portion of the pie he values more gets destroyed with every period of disagreement. This increases player 2’s bargaining power, allowing him to extract a larger fraction of the surplus.

When $\varphi \leq \varphi(r, \rho)$, the share that player 1 consumes of pie $X$ is decreasing in $\varphi$ and increasing in both $r$ and $\rho$. As $\varphi$ increases, player 2 bargains from a stronger position, since it becomes less costly for him to delay an agreement. When $r$ is close to 1, the
Table 1. Consumption Shares of Pies X and Y

<table>
<thead>
<tr>
<th>Complete Offers</th>
<th>Partial Offers</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t = 0 )</td>
<td>( t = t_Y )</td>
</tr>
<tr>
<td>( \phi \leq \bar{\phi}(r, \rho) )</td>
<td>( \phi &gt; \bar{\phi}(r, \rho) )</td>
</tr>
<tr>
<td>( x_1^1 = x_2^1 = \frac{\phi(2r-1)+\rho}{2r(\phi+\rho)} )</td>
<td>( \frac{\phi(2r-1)+\rho}{2r(\phi+\rho)} )</td>
</tr>
<tr>
<td>( x_1^2 = x_2^2 = \frac{\phi+r}{2r(\phi+\rho)} )</td>
<td>( \frac{\phi+r}{2r(\phi+\rho)} )</td>
</tr>
<tr>
<td>( y_1^1 = y_1^2 = - \frac{1}{2} )</td>
<td>( - \frac{1}{2} )</td>
</tr>
<tr>
<td>( y_2^1 = y_2^2 = - \frac{1}{2} )</td>
<td>( - \frac{1}{2} )</td>
</tr>
<tr>
<td>( t = 0 )</td>
<td>( t = t_Y )</td>
</tr>
<tr>
<td>( \phi \leq \bar{\phi}(r, \rho) )</td>
<td>( \phi &gt; \bar{\phi}(r, \rho) )</td>
</tr>
<tr>
<td>( x_1^1 = x_2^1 = \frac{(1-r)(2\phi+\rho)}{2(\phi+\rho)} )</td>
<td>( \frac{(1-r)(2\phi+\rho)}{2(\phi+\rho)} )</td>
</tr>
<tr>
<td>( x_1^2 = x_2^2 = \frac{(1-r)r}{2(\phi+\rho)} )</td>
<td>( \frac{(1-r)r}{2(\phi+\rho)} )</td>
</tr>
<tr>
<td>( y_1^1 = y_1^2 = - \frac{1}{2} )</td>
<td>( - \frac{1}{2} )</td>
</tr>
<tr>
<td>( y_2^1 = y_2^2 = - \frac{1}{2} )</td>
<td>( - \frac{1}{2} )</td>
</tr>
</tbody>
</table>

The case of complete offers is divided into the subcases \( \phi \leq \bar{\phi}(r, \rho) \) and \( \phi > \bar{\phi}(r, \rho) \). The columns \( t = 0 \) indicate proposals that are accepted in the first period, while the \( t = t_Y \) columns indicate proposals that are accepted in the period that pie \( Y \) arrives. Similarly for the case of partial offers.

Asymmetries in valuation are small and therefore the cost to player 1 for delaying an agreement is small. Finally, when \( \rho \) is large the benefit that player 2 gets from delaying an agreement is small, since waiting for pie \( Y \) is more costly.

Next, we turn to the case in which agents are able to make partial offers, as in Section 4. We know by Theorem 2 that for any \( \Delta > 0 \) agents will come to a complete agreement over \( X \) only if \( p \leq \bar{p}(\delta, r_1, r_2) \), which is equivalent to

\[
\frac{p(\Delta)}{1 - \delta(\Delta)} \leq \frac{2r^2}{1 - r^2} \frac{1}{\delta(\Delta)},
\]

and will come to a partial agreement if the reverse inequality holds, consuming fraction \( 1 - \hat{\lambda}(\Delta) = 1 - \frac{r\delta(\Delta)}{2 - \delta(\Delta)} \) of pie \( X \). Replacing \( \delta(\Delta) = e^{-\rho\Delta} \) and \( p(\Delta) = 1 - e^{-\varphi\Delta} \) in the above expression and taking limits as \( \Delta \to 0 \) we get that in the continuous time limit agents will come to a complete agreement over \( X \) at the beginning of the game only if

\[
\varphi \leq \hat{\phi}(r, \rho) = \frac{2r^2\rho}{1 - r^2},
\]

in which case the consumption shares are exactly the same as in the case of complete offers when \( \phi \leq \bar{\phi}(r, \rho) \). If the reverse inequality holds, then in the continuous time limit the players come to a partial agreement, consuming fraction

\[
\lim_{\Delta \to 0} \left( 1 - \hat{\lambda}(\Delta) \right) = 1 - r
\]

of pie \( X \) at date 0, and the remaining fraction \( r \) as soon as pie \( Y \) arrives. Moreover, when pie \( Y \) arrives, player 1 will consume all of what’s left of pie \( X \) and player 2 will
consume all of pie $Y$ (this follows from equation (A10) in the appendix). The exact division of the fraction $1 - r$ of pie $X$ that is consumed in the beginning of the game is reported in Table 1. Notice that player 1 consumes a larger share of pie $X$ than player 2 at the beginning of the game. The reason for this is that, although the costs of delaying an agreement are still larger for player 1 than they are for player 2, the benefit that player 2 gets from delaying is substantially reduced when the players can reach partial agreements, as in this case player 2 will still consume all of pie $Y$ as soon as it arrives.

Also notice that the share of pie $X$ that player 1 gets at the beginning of the game is now increasing in $\varphi$ and decreasing in $\rho$. In other words, the comparative statics now run in the opposite direction. When players reach a partial agreement, player 2 gets a share of pie $X$ at the beginning of the game and all of pie $Y$ as soon as it arrives. In this case, player 1 always has the option of delaying an agreement until the second pie arrives, in which case he consumes all of pie $X$. When the costs of waiting are small (either because $\varphi$ is large or because $\rho$ is small) player 1 bargains from a strong position, as now the value of delaying is large.

6 Applications

In this section, we discuss two possible applications to the models we developed in the previous sections. The discussions are not meant to depict perfect applications of our theory, but rather to raise new questions and outline areas of related future work.

6.1 Logrolling

Throughout the first year of the Obama presidency, the Democratic leadership of both the House and Senate were hard-pressed to generate support for their legislation from even within their party, let alone persuade a Republican member of Congress to endorse their proposals. For example, the first version of the healthcare bill came close to not making it through the Senate, while the version that finally passed required an accompanying reconciliation measure and barely made it through the House.

Among the many reasons why the negotiations were so intense, what stands out to us is the fact that many issues that are traditionally important to moderate and conservative lawmakers were notably absent from the bargaining table. It is interesting to consider the possibility that had fiscal policy or tort reform also been on the table during the healthcare negotiations, perhaps it would not have been difficult for the Democrats
to find support from either the more conservative members of their party or even from some Republican members of Congress. The absence of these issues could be interpreted as a timing failure, in line with our assumption that there is a commitment problem arising from an exogenous constraint on when an issue can be resolved. Admittedly, one could argue that there were no exogenous constraints to putting tort reform on the table (though this is debatable); but in a deep recession that called for an increase in public spending, the Democrats surely did not consider fiscal restraint to even be a possibility. Could the liberal Democrats have persuaded the more fiscally conservative members of their coalition that as soon as the recession cleared, they would support a reduction in government spending in exchange for the conservatives’ votes on the healthcare bill? If such promises are not credible, then the insights of our model may apply.

Whether the bill that passed in Congress is a partial or full agreement between those who voted for it is a matter of debate. However, there are reasons to believe that the bill that was finally signed into law is an incomplete agreement. The bill does not do much to address the rising costs of medical care (a consequence of the current payment system), and this is an issue that virtually all members of Congress agree must imminently be addressed. Therefore, some may view the recent piece of legislation as only a partial agreement.

6.2 International Trade Negotiations

In July 2008, trade negotiations that began in Doha in 2001 broke down as a result of an impasse caused by disagreements between developed and less developed countries (LDCs) over agricultural subsidies. The LDCs have taken the position that these subsidies, provided by the governments of many EU countries and the United States to their farmers, effectively act as trade barriers and are equivalent to tariffs on agricultural imports. In principle, the developed countries could offer to liberalize their agriculture policies in exchange for an agreement on industrial goods and intellectual property that is favorable to them (Schnepf and Hanrahan 2009). As it stands, the agricultural issue appears to be more important for many of the LDCs than issues related to industrial goods or intellectual property rights. On the other hand, the negotiators on behalf of the developed countries appear to have the reverse priorities.

To liberalize their agricultural policies, however, the governments of the developed countries must first overcome the powerful farmers’ lobby, and they must convince leg-

---

9India may be a notable exception.
islators from largely agrarian voting districts to vote on liberal policies. This takes both time and effort; in particular, a promise to do so gradually, or in the future, may not be credible.

Our model captures some of the salient features of the Doha conundrum. If the negotiating parties could write enforceable contracts, then delays such as the present one (following the Cancun talks), could potentially be avoided. But enforceable contracts are not always available in international negotiations between states. There are also exogenous constraints on the timing of the Doha negotiations that create incentives comparable to those that arise in our model. For example, the talks are structured in a way that makes it difficult to proceed without coming to a comprehensive agreement on multiple issues.

7 Conclusion

We constructed a simple model of bargaining to show that when two players negotiate over multiple issues, then in the absence of fully contingent contracts the timing of the issues matters. Our model stresses the link between the commitment problem and inefficiencies arising from delay. We showed that commitment problems may lead to delays when players have asymmetric preferences over the issues. We therefore gave a new explanation to why parties may fail to reach an early agreement even when delay is costly.

We then extended our model to the case in which the players were able to make partial offers over the existing pie, leaving the remainder for future negotiations. In this setup, we found that in some circumstances the players reach a partial agreement on the first issue, and complete the agreement only when the second issue is on the table. Put differently, we found that in this case, agents may consume the first pie in two stages: a fraction of it in the first period, and the remainder when the second pie arrives.

Finally, we showed that the main results of our model continue to hold even as we take the time interval between offers to 0. That is, the model continues to deliver delay and partial agreements even as bargaining frictions vanish. We ended with a discussion of two applications. Though the applications do not fit the model perfectly, they raise new questions that we hope will be answered in future work.
8 Appendix

8.1 Uniqueness of Equilibrium Payoffs in Theorem 1

We prove the first statement of Theorem 1 that the equilibrium payoffs are unique and can be supported by a profile of stationary strategies. First note that any subgame that starts either when the state is $Y$ or when the state is $XY$ is a classic bilateral bargaining game. Hence, one can apply an argument similar to the one pioneered by Rubinstein (1982) to show that there are unique equilibrium payoffs. Let $v_i(j, s)$ be the (unique) equilibrium payoffs of player $i$ in state $(j, s)$ for $s = Y$ or $XY$. In state $\emptyset$ there is no consumption of either pie, so

$$w_i(\emptyset) = \delta (pw_i(Y) + (1 - p)w_i(\emptyset))$$
$$\Rightarrow w_i(\emptyset) = \alpha w_i(Y), \quad \text{where} \quad \alpha = \frac{\delta p}{1 - \delta(1 - p)} \quad (A1)$$

discourts the consumption of $Y$ according to when the pie is expected to arrive. We begin by showing that if there is an equilibrium then the payoffs in state $X$ are unique.

Assume that the set of subgame perfect equilibria is nonempty. Let $\overline{v}_i(j, X)$ and $\underline{v}_i(j, X)$ denote the supremum and infimum subgame perfect equilibrium payoffs of player $i$ in state $(j, X)$. Define

$$\overline{w}_i(X) = \frac{1}{2} \overline{v}_i(1, X) + \frac{1}{2} \overline{v}_i(2, X)$$
$$\underline{w}_i(X) = \frac{1}{2} \underline{v}_i(1, X) + \frac{1}{2} \underline{v}_i(2, X). \quad (A2)$$

To show that $\overline{v}_i(j, X) = \underline{v}_i(j, X)$ it suffices to show that $\overline{w}_i(X) = \underline{w}_i(X)$. Using standard arguments, $\overline{v}_i(1, X)$ and $\overline{v}_i(2, X)$ satisfy

$$\overline{v}_i(1, X) \leq \max \left\{ \frac{1 + \frac{w_2(\emptyset)}{r_2} - \frac{\delta}{r_2} (pw_2(XY) + (1 - p)w_2(X)) + w_1(\emptyset)}{\delta (pw_1(XY) + (1 - p)\overline{w}_1(X))} \right\}$$
$$\overline{v}_i(2, X) \leq \frac{\delta (pw_1(XY) + (1 - p)\overline{w}_1(X))}{w_1(\emptyset)}$$

Combining these two inequalities yields

$$\overline{w}_1(X) \leq \frac{1}{2} \max \left\{ \frac{1 + \frac{w_2(\emptyset)}{r_2} - \frac{\delta}{r_2} (pw_2(XY) + (1 - p)w_2(X)) + w_1(\emptyset)}{\delta (pw_1(XY) + (1 - p)\overline{w}_1(X))} \right\}$$
$$+ \frac{1}{2} \frac{\delta (pw_1(XY) + (1 - p)\overline{w}_1(X))}{w_1(\emptyset)}, \quad (A3)$$
By a similar argument, we get
\[ w_1(X) \geq \frac{1}{2} \max \left\{ 1 + \frac{w_{2}(q)}{r_2} - \frac{\delta}{r_2} \left( p w_2(XY) + (1-p) w_2(X) \right) + w_1(0), \right\} \]
\[ + \frac{1}{2} \delta (p w_1(XY) + (1-p) w_1(X)) \] (A4)

Finally, substracting (A4) from (A3) we get
\[ w_1(X) - w_1(X) \leq \frac{1}{2} \delta (1-p) \max \left\{ \frac{1}{r_2} (w_2(X) - w_2(X)), (w_1(X) - w_1(X)) \right\} \]
\[ + \frac{1}{2} \delta (1-p) (w_1(X) - w_1(X)). \] (A5)

A symmetric argument for player 2 establishes
\[ w_2(X) - w_2(X) \leq \frac{1}{2} \delta (1-p) \max \left\{ r_2 (w_1(X) - w_1(X)), (w_2(X) - w_2(X)) \right\} \]
\[ + \frac{1}{2} \delta (1-p) (w_2(X) - w_2(X)). \] (A6)

There are two possible cases:

1. \( r_2 (w_1(X) - w_1(X)) \geq (w_2(X) - w_2(X)), \) and
2. \( r_2 (w_1(X) - w_1(X)) \leq (w_2(X) - w_2(X)). \)

In case (1), from (A5) we have
\[ w_1(X) - w_2(X) \leq \delta (1-p) (w_1(X) - w_1(X)) \Rightarrow w_1(X) - w_2(X) = 0. \]

Since \( r_2 (w_1(X) - w_1(X)) \geq (w_2(X) - w_2(X)) \geq 0 \) and \( r_2 > 0 \) we have that \( w_2(X) - w_2(X) = 0. \) In case (2), from (A6) we have
\[ w_2(X) - w_2(X) \leq \delta (1-p) (w_2(X) - w_2(X)) \Rightarrow w_2(X) - w_2(X) = 0. \]

Since \( w_2(X) - w_2(X) \geq r_2 (w_1(X) - w_1(X)) \geq 0 \) and \( r_2 > 0 \) we have \( w_1(X) - w_1(X) = 0. \) We conclude that \( w_i(X) = v_i(X), \) and hence \( v_{ij}(j, X) = v_j(j, X) \) for all \( i, j. \)

Finally, we show that the set of subgame perfect equilibria is nonempty. We know that any subgame that starts at states XY or Y has a unique equilibrium. Therefore, we now present strategies for the players for state X which, together with their strategies at states XY and Y, form a subgame perfect equilibrium of the game.

Let \( v_i(j, X) \) be the unique candidates for subgame perfect payoffs, and let \( w_i(X) = \frac{1}{2} v_i(1, X) + \frac{1}{2} v_i(2, X). \) Consider the following strategy profile (for state X): in state


\((j, X)\) \(i\) accepts an offer if and only if accepting gives him a total payoff of at least \(\delta (pw_i(XY) + (1 - p)w_i(X))\). If the state is \((1, X)\), and

\[
1 + \frac{w_2(\emptyset)}{r_2} + w_1(\emptyset) \geq \delta (pv_1(XY) + (1 - p)w_1(X)) + \frac{\delta}{r_2} (pw_2(XY) + (1 - p)w_2(X))
\]

then player 1 offers shares \(\{1 - x_2^1, x_1^1\}\) where \(r_2 x_2^1 + w_2(\emptyset) = \delta (pw_2(XY) + (1 - p)w_2(X))\); otherwise, he offers \(\{1, 0\}\). Similarly, if the above inequality holds then in state \((2, X)\) player 2 offers division \(\{x_1^1, 1 - x_1^1\}\) with \(x_1^1 + w_1(\emptyset) = \delta (pw_1(XY) + (1 - p)w_1(X))\); otherwise, he offers \(\{0, 1\}\). One can check that there are no profitable one-shot deviations. Therefore, these strategies (together with the equilibrium strategies at states \(XY\) and \(Y\)) form a subgame perfect equilibrium.

### 8.2 Proof of Theorem 2

**Lemma 1** In any equilibrium, the players reach a complete agreement when the state is \(\lambda XY\) with \(\lambda \in [0, 1]\). Moreover, the equilibrium payoffs for these states are unique.

**Proof.** Omitted; the proof is available upon request from the authors. ■

We use Lemma 1 to characterize the equilibrium offers and value functions in states \(\lambda XY\). Let the state be \(\lambda XY\) for any \(\lambda \in [0, 1]\). Denote by \(\{x_1^j, x_2^j, y_1^j, y_2^j\}\) the consumption shares proposed by player \(j\) when he is proposer. By Lemma 1, players come to a complete agreement in all states \(\lambda XY\). Therefore, the shares must solve:

\[
\begin{align*}
\max_{x_1^1, x_2^1, y_1^1, y_2^1 \geq 0} & \quad x_1^1 + r_1 y_1^1 \\
\text{subject to} & \quad r_2 x_2^1 + y_2^1 \geq \delta \left( \frac{1}{2} (r_2 x_2^1 + y_2^1) + \frac{1}{2} (r_2 x_2^2 + y_2^2) \right) \\
& \quad x_1^1 + x_2^1 \leq \lambda, \ y_1^1 + y_2^1 \leq 1
\end{align*}
\tag{A7}
\]

\[
\begin{align*}
\max_{x_1^2, x_2^2, y_1^2, y_2^2 \geq 0} & \quad r_2 x_2^2 + y_2^2 \\
\text{subject to} & \quad x_1^2 + r_1 y_1^2 \geq \delta \left( \frac{1}{2} (x_1^2 + r_1 y_1^2) + \frac{1}{2} (x_1^1 + r_1 y_1^1) \right) \\
& \quad x_1^2 + x_2^2 \leq \lambda, \ y_1^2 + y_2^2 \leq 1
\end{align*}
\tag{A8}
\]
The solution to these programs are as follows: If \( \lambda \geq \hat{\lambda}(\delta, r_1) = \frac{r_1 \delta}{1 - \delta} \), then

\[
((x_1^1, x_2^1), (y_1^1, y_2^1)) = \left( (\lambda, 0), \left( \frac{2(1 - \delta)(2 - \delta(1 + r_2 \lambda))}{4(1 - \delta) + \delta^2(1 - r_1 r_2)}, 1 - \frac{2(1 - \delta)(2 - \delta(1 + r_2 \lambda))}{4(1 - \delta) + \delta^2(1 - r_1 r_2)} \right) \right)
\]

\[
((x_1^2, x_2^2), (y_1^2, y_2^2)) = \left( 1 - \frac{2(1 - \delta)(\lambda(2 - \delta) - r_1 \delta)}{4(1 - \delta) + \delta^2(1 - r_1 r_2)}, \frac{2(1 - \delta)(\lambda(2 - \delta) - r_1 \delta)}{4(1 - \delta) + \delta^2(1 - r_1 r_2)}, (0, 1) \right)
\]

(A9)

Otherwise, for \( \lambda \leq \hat{\lambda}(\delta, r_1) \),

\[
((x_1^1, x_2^1), (y_1^1, y_2^1)) = \left( (\lambda, 0), \left( 1 - \frac{\delta(r_1 + \lambda)}{2r_1}, \frac{\delta(r_1 + \lambda)}{2r_1} \right) \right)
\]

\[
((x_1^2, x_2^2), (y_1^2, y_2^2)) = \left( (\lambda, 0), \left( \frac{r_1 \delta - \lambda(2 - \delta)}{2r_1}, 1 - \frac{r_1 \delta - \lambda(2 - \delta)}{2r_1} \right) \right)
\]

(A10)

The value functions are obviously given by \( v_1(j, \lambda XY) = x_1^j + ry_1^j \) and \( v_2(j, \lambda XY) = rx_2^j + ry_2^j \) for both cases. We calculate

\[
w_1(\lambda XY) = \begin{cases} 
\frac{2r_1(1 - \delta) + \lambda(2 - \delta - r_1 r_2 \delta)}{4(1 - \delta) + \delta^2(1 - r_1 r_2)} & \text{when } \lambda > \hat{\lambda} \\
\frac{r_1 + \lambda}{2r_1} & \text{when } \lambda \leq \hat{\lambda}
\end{cases}
\]

\[
w_2(\lambda XY) = \begin{cases} 
\frac{(2 - \delta - r_1 r_2 \delta) + 2r_2 \lambda(1 - \delta)}{4(1 - \delta) + \delta^2(1 - r_1 r_2)} & \text{when } \lambda > \hat{\lambda} \\
\frac{r_1 + \lambda}{2r_1} & \text{when } \lambda \leq \hat{\lambda},
\end{cases}
\]

where we have dropped the argument of \( \hat{\lambda} \). Define \( W(\lambda XY) = r_2 w_1(\lambda XY) + w_2(\lambda XY) \).

Now we return to the proof of Theorem 2. Recall that \( \alpha = \frac{\delta p}{1 - \delta(1 - p)} \) and note that \( \alpha \leq 2r_1 r_2/(1 + r_1 r_2) \) can be rewritten

\[
p \leq \hat{\rho}(\delta, r_1, r_2)
\]

\[
= \frac{2r_1 r_2}{1 - r_1 r_2} \frac{1 - \delta}{\delta}.
\]

Take any subgame beginning in state \( \lambda X \) and denote the period in which that subgame begins by period 0 (wlog this is just a renumbering). A consumption path is a sequence \( \{x_1, x_2\}_0^\infty \) with the interpretation that \( x_{it} \) is the share of \( X \) that player \( i \) consumes in period \( t \) conditional on the event that \( Y \) has not arrived by period \( t \). For a consumption path \( \{x_1, x_2\}_0^\infty \) define the associated consumption sequence \( \{\mu_t\}_0^\infty \) to be a sequence with \( \mu_t = x_{1t} + x_{2t} \) for all \( t \). Each \( (x_1t, x_2t) \) is history-dependent and there may be several possible realizations for any equilibrium; in particular, any \( (x_1t, x_2t) \) may
depend on the stochastic elements of the history up to period $t$. The total payoff of the players from any consumption path $\{x_{1t}, x_{2t}\}_0^\infty$ is defined as

$$E \left[ r_2 \sum_{t=0}^\infty \delta^t u_1(x_{1t}, y_{1t}) + \sum_{t=0}^\infty \delta^t u_2(x_{2t}, y_{2t}) \right]$$

where the $x_{it}$’s are given by the consumption path $\{x_{1t}, x_{2t}\}_0^\infty$ until pie $Y$ arrives, and are determined in equilibrium along with the $y_{it}$’s after $Y$ arrives. One can show that for a given consumption path $\{x_{1t}, x_{2t}\}_0^\infty$ starting at state $\lambda X$, the total payoff equals

$$r_2 \sum_{t=0}^\infty \delta^t (1-p)^t \mu_t + \delta p \sum_{t=0}^\infty \left[ \delta^t (1-p)^t W \left( \left( \lambda - \sum_{t'=0}^t \mu_{t'} \right) XY \right) \right]$$

where $\{\mu_t\}$ is the consumption sequence associated with the path $\{x_{1t}, x_{2t}\}_0^\infty$ and for any $\gamma \in [0, 1], W(\gamma XY) = r_2 w_1(\gamma XY) + w_2(\gamma XY)$. Note that the total payoff depends only on $\{\mu_t\}_0^\infty$ and not on what each player consumes. The results for states $\lambda X$ are proven next.

**Lemma 2** If $p > \hat{p}(\delta, r_1, r_2)$, then in any equilibrium the players delay in states $\lambda X$ when $\lambda \leq \hat{\lambda}$.

**Proof.** Let $\{\mu_t\}_0^\infty$ be an equilibrium consumption sequence, and assume for the sake of contradiction that $\lambda \geq \mu_0 > 0$. The total payoff from this consumption sequence is

$$r_2 \sum_{t=0}^\infty \delta^t (1-p)^t \mu_t + \delta p \sum_{t=0}^\infty \left[ \delta^t (1-p)^t W \left( \left( \lambda - \sum_{t'=0}^t \mu_{t'} \right) XY \right) \right]$$

$$= \left[ r_2 - \alpha \left( \frac{r_2}{2} + \frac{1}{2r_1} \right) \right] \sum_{t=0}^\infty \delta^t (1-p)^t \mu_t + \alpha W(\lambda XY) < \alpha W(\lambda XY),$$

where the equality follows from substituting the terms $W(\gamma XY)$ and the inequality follows from the fact that $p > \hat{p}(\delta, r_1, r_2)$ implies that $\left[ r_2 - \alpha \left( \frac{r_2}{2} + \frac{1}{2r_1} \right) \right] < 0$. Therefore, the total payoff from any strategy profile that creates a realization $\{\mu_t\}_0^\infty$ with $\mu_0 > 0$ is strictly less than the total payoff to waiting for $Y$ to arrive. This means that at least one of the two players is getting a payoff strictly less than he would if there were delay. Since that player can unilaterally deviate to generate delay (by either rejecting a proposal or making an outrageous offer that the other player will reject), it cannot be that the players consume $\mu_0 > 0$ in states $\lambda X$ when $\lambda \leq \hat{\lambda}$. ■
Lemma 3 If \( p > \hat{p}(\delta, r_1, r_2) \) then in any state \( \lambda X \) with \( \lambda > \hat{\lambda} \), the total payoff is uniquely maximized by the consumption sequence \( \{\mu_t\}_0^\infty \) with \( \mu_0 = \lambda - \hat{\lambda} \) and \( \mu_t = 0 \) for all \( t > 0 \).

Proof. Fix a consumption sequence \( \{\mu_t\}_0^\infty \). By Lemma 2, if \( \{\mu_t\}_0^\infty \) is such that \( \sum_{t=0}^t \mu_t \geq \lambda - \hat{\lambda} \) for some \( t' \geq 0 \), then the consumption sequence \( \{\mu'_t\}_0^\infty \) with \( \mu'_t = \mu_t \) for \( t \leq t' \) and \( \mu'_t = 0 \) for \( t > t' \) will give the players a larger total payoff than \( \{\mu_t\}_0^\infty \).

Next, we show that the total payoff from consuming \( \mu_0 > \lambda - \hat{\lambda} \) is less than the total payoff from consuming only \( \lambda - \hat{\lambda} \). By Lemma 2 players will delay agreement in any state \( \lambda X \) such that \( \lambda \leq \hat{\lambda} \). Therefore,

\[
\begin{align*}
    r_2\mu_0 + \alpha \left[ r_2 \left( \frac{r_1 + \lambda - \mu_0}{2} \right) + \frac{r_1 + \lambda - \mu_0}{2r_1} \right] < r_2(\lambda - \hat{\lambda}) + \alpha \left[ r_2 \left( \frac{r_1 + \hat{\lambda}}{2} \right) + \frac{r_1 + \hat{\lambda}}{2r_1} \right],
\end{align*}
\]

which follows because \( p > \hat{p}(\delta, r_1, r_2) \). Therefore, \( \{\mu_t\}_0^\infty \) maximizes total payoff it must be that \( \mu_0 \leq \lambda - \hat{\lambda} \). Suppose \( \mu_0 < \lambda - \hat{\lambda} \). Then, by the same argument, if \( \{\mu_t\}_0^\infty \) maximizes total payoff it must be that \( \mu_1 \leq \lambda - \mu_0 - \hat{\lambda} \), or \( \mu_0 + \mu_1 \leq \lambda - \hat{\lambda} \). Repeating this argument inductively we conclude that if \( \{\mu_t\}_0^\infty \) maximizes total payoff, it must be that \( \sum_{t=0}^\infty \mu_t \leq \lambda - \hat{\lambda} \).

Finally, we show that the total payoff is maximized by setting \( \mu_0 = \lambda - \hat{\lambda} \) and \( \mu_t = 0 \) for all \( t > 0 \). To see this, fix a consumption sequence \( \{\mu_t\}_0^\infty \) with \( \mu_0 < \lambda - \hat{\lambda} \) and \( \sum_{t=0}^\infty \mu_t \leq \lambda - \hat{\lambda} \). Then we have

\[
\begin{align*}
    r_2 \sum_{t=0}^\infty \delta^t (1-p)^t \mu_t + \delta \sum_{t=0}^\infty \left[ \delta^t (1-p)^t W \left( \left( \lambda - \sum_{\tau=0}^t \mu_t \right) XY \right) \right] < r_2(\lambda - \hat{\lambda}) + \alpha \left[ r_2 \left( \frac{r_1 + \hat{\lambda}}{2} \right) + \frac{r_1 + \hat{\lambda}}{2r_1} \right],
\end{align*}
\]

which follows from substituting the terms \( W(\gamma XY) \) and noting that \( \sum_{t=0}^\infty \delta^t (1-p)^t \mu_t < \lambda - \hat{\lambda} \). ■

Lemma 4 Let \( \underline{v}_i(j, \lambda X) \) and \( \overline{v}_i(j, \lambda X) \) be the infimum and supremum subgame perfect equilibrium payoffs for player \( i \) when the state is \( (j, \lambda X) \), and define \( \underline{w}_i(\lambda X) \) and \( \overline{w}_i(\lambda X) \) as in (A2). If \( \lambda > \hat{\lambda} \) and \( p > \hat{p}(\delta, r_1, r_2) \), then the following are true.
(i) \( \varphi_i(j, \lambda X) \geq \delta(pw_i(\lambda XY) + (1 - p)\overline{w}_i(\lambda X)) \) for all \( i \) and \( j \neq i \)

(ii) \( \varphi_i(j, \lambda X) \leq \delta(pw_i(\lambda XY) + (1 - p)\overline{w}_i(\lambda X)) \) for all \( i \) and \( j \neq i \)

(iii) \( r_2\varphi_1(1, \lambda X) \geq r_2(\lambda - \hat{\lambda}) + \alpha W(\hat{\lambda}XY) - \delta(pw_2(\lambda XY) + (1 - p)\overline{w}_2(\lambda X)) \)

(iv) \( r_2\varphi_1(1, \lambda X) \leq r_2(\lambda - \hat{\lambda}) + \alpha W(\hat{\lambda}XY) - \delta(pw_2(\lambda XY) + (1 - p)\overline{w}_2(\lambda X)) \)

(v) \( \psi_2(2, \lambda X) \geq r_2(\lambda - \hat{\lambda}) + \alpha W(\hat{\lambda}XY) - r_2\delta(pw_1(\lambda XY) + (1 - p)\overline{w}_1(\lambda X)) \)

(vi) \( \overline{\psi}_2(2, \lambda X) \leq r_2(\lambda - \hat{\lambda}) + \alpha W(\hat{\lambda}XY) - r_2\delta(pw_1(\lambda XY) + (1 - p)\overline{w}_1(\lambda X)) \)

**Proof.** Claims (i) and (ii) are immediate, and the arguments for (v) and (vi) are essentially the same as the arguments for (iii) and (iv). We therefore only prove (iii) and (iv). We start with (iii).

Suppose the state is \( \lambda X \) with \( \lambda > \hat{\lambda} \). If player 1 offers player 2 a fraction \( \overline{x}_2 \) of pie \( X \) such that

\[
r_2\overline{x}_2 + \alpha w_2(\hat{\lambda}XY) = \delta(pw_2(\lambda XY) + (1 - p)\overline{w}_2(\lambda X))
\]

then the offer will be accepted. Moreover, we can show that \( \overline{x}_2 \leq \lambda - \hat{\lambda} \), so that \( \overline{x}_2 \) is a feasible offer. To see this, note that Lemma 3 implies

\[
\overline{w}_2(\lambda X) + r_2\overline{w}_1(\lambda X) \leq r_2(\lambda - \hat{\lambda}) + \alpha W(\hat{\lambda}XY).
\]

Note that \( \overline{w}_1(\lambda X) \geq \alpha w_1(\lambda XY) \) as player 1 can always guarantee this much. Using this together with the definition of \( W(\hat{\lambda}XY) \) we get

\[
\overline{w}_2(\lambda X) \leq r_2(\lambda - \hat{\lambda}) + \alpha w_2(\hat{\lambda}XY') + \alpha r_2\left[w_1(\hat{\lambda}XY) - w_1(\lambda XY)\right]
\]

\[
< r_2(\lambda - \hat{\lambda}) + \alpha w_2(\hat{\lambda}XY'),
\]

where the second inequality follows from the fact that \( \lambda > \hat{\lambda} \) and that \( w_1(\gamma XY) \) is increasing in \( \gamma \). Therefore, to show that \( \overline{x}_2 \leq \lambda - \hat{\lambda} \) it suffices to show that

\[
r_2(\lambda - \hat{\lambda}) + \alpha w_2(\hat{\lambda}XY') - \delta\left(pw_2(\lambda XY) + (1 - p)\left[r_2(\lambda - \hat{\lambda}) + \alpha w_2(\hat{\lambda}XY')\right]\right) \geq 0
\]

which follows after replacing the expressions for \( w_2(\hat{\lambda}XY') \) and \( w_2(\lambda XY) \).

Since player 2 must accept any offer that gives him \( \overline{x}_2 \) (and since \( \overline{x}_2 \leq \lambda - \hat{\lambda} \), it must be that

\[
r_2\varphi_1(1, \lambda X) \geq r_2\left(\lambda - \hat{\lambda} - \overline{x}_2 + \alpha w_1(\hat{\lambda}XY')\right)
\]

\[
= r_2(\lambda - \hat{\lambda}) - \delta(pw_2(\lambda XY) + (1 - p)\overline{w}_2(\lambda X)) + \alpha\left(r_2w_1(\hat{\lambda}XY') + w_2(\hat{\lambda}XY')\right)
\]

24
which establishes (iii).

Next, we prove (iv). Note that the lowest offer that player 2 will accept in state $\lambda X$ is one that gives him a payoff $\delta (pw_2(\lambda XY) + (1 - p)w_2(\lambda X))$. Let $x_2$ be such that

$$r_2x_2 + \alpha w_2(\hat{\lambda}XY) = \delta (pw_2(\lambda XY) + (1 - p)w_2(\lambda X)).$$

If player 2 accepts an offer that gives him a payoff of $\delta (pw_2(\lambda XY) + (1 - p)w_2(\lambda X))$, then Lemma 3 implies that player 1’s payoff is maximized by offering consumption shares \{\lambda - \hat{\lambda} - x_2, x_2\}. Note that $x_2$ is feasible, since $x_2 \leq \tau_2 \leq \lambda - \hat{\lambda}$ (which follows from $\overline{w}_2(\lambda X) \geq w_2(\lambda X)$). Therefore, we have

$$r_2\overline{v}_1(1, \lambda X) \leq r_2 \left( \lambda - \hat{\lambda} - x_2 + \alpha w_1(\lambda XY) \right)$$

$$= r_2 \left( \lambda - \hat{\lambda} \right) - \delta (pw_2(\lambda XY) + (1 - p)w_2(\lambda X)) + \alpha \left( r_2w_1(\hat{\lambda}XY) + w_2(\hat{\lambda}XY) \right)$$

which establishes (iv). ■

**Lemma 5** Let the state be $\lambda X$ with $\lambda > \hat{\lambda}$. If $p > \hat{p}(\delta, r_1, r_2)$, then in any equilibrium the players reach a partial agreement, consuming $x_1 + x_2 = \lambda - \hat{\lambda}$.

**Proof.** The inequalities stated in Lemma 4 imply

$$r_2\overline{w}_1(\lambda X) \leq \frac{1}{2} \left( r_2 \left( \lambda - \hat{\lambda} \right) + \alpha W(\hat{\lambda}) - \delta (pw_2(\lambda XY) + (1 - p)w_2(\lambda X)) \right)$$

$$+ \frac{1}{2} r_2\delta (pw_1(\lambda XY) + (1 - p)\overline{w}_1(\lambda X))$$

$$r_2w_1(\lambda X) \geq \frac{1}{2} \left( r_2 \left( \lambda - \hat{\lambda} \right) + \alpha W(\hat{\lambda}) - \delta (pw_2(\lambda XY) + (1 - p)\overline{w}_2(\lambda X)) \right)$$

$$+ \frac{1}{2} r_2\delta (pw_1(\lambda XY) + (1 - p)\overline{w}_1(\lambda X)).$$

These in turn imply

$$r_2 \left( \overline{w}_1(\lambda X) - \overline{w}_1(\lambda X) \right) \leq \frac{1}{2} (1 - p) \left( \overline{w}_2(\lambda X) - w_2(\lambda X) + \delta r_2 (\overline{w}_1(\lambda X) - \overline{w}_1(\lambda X)) \right)$$

$$\Rightarrow r_2 \left( \overline{w}_1(\lambda X) - w_1(\lambda X) \right) \leq \frac{(1 - p) \left( \overline{w}_2(\lambda X) - w_2(\lambda X) \right)}{2 - \delta(1 - p)}. \quad (A11)$$

Similarly, for player 2 we get

$$\overline{w}_2(\lambda X) - w_2(\lambda X) \leq \frac{r_2(1 - p) \left( \overline{w}_1(\lambda X) - w_1(\lambda X) \right)}{2 - \delta(1 - p)}. \quad (A12)$$

25
Combining (A11) and (A12),
\[ \bar{w}_1(\lambda X) - w_1(\lambda X) \leq \left( \frac{1 - p}{2 - \delta(1 - p)} \right)^2 (\bar{w}_1(\lambda X) - w_1(\lambda X)). \] (A13)

But this implies \( \bar{w}_1(\lambda X) = w_1(\lambda X) \). Then by (A12) we also have \( \bar{w}_2(\lambda X) = w_2(\lambda X) \).

Substituting back, we find that for \( i \neq j \)
\[
\begin{align*}
  v_i(j, \lambda X) &= \delta (pw_i(\lambda XY) + (1 - p)w_i(\lambda X)) \\
  r_2v_1(1, \lambda X) &= r_2 \left( \lambda - \hat{\lambda} \right) + \alpha W(\hat{\lambda}XY) - w_2(\lambda X) \\
  v_2(2, \lambda X) &= r_2 \left( \lambda - \hat{\lambda} \right) + \alpha W(\hat{\lambda}XY) - r_2w_1(\lambda X)
\end{align*}
\]

Finally, note that these payoffs can only be supported by a strategy profile such that the proposer makes an offer to consume a total of \( \lambda - \hat{\lambda} \) and the responder accepts. ■

**Lemma 6** If \( p < \hat{p}(\delta, r_1, r_2) \), then the players reach a complete agreement (consuming \( x_1 + x_2 = \lambda \)) in all states \( \lambda X \).

**Proof. (sketch only)** Following a similar argument to that used in Lemma 2, we can show that if \( p < \hat{p}(\delta, r_1, r_2) \) and \( \lambda \leq \hat{\lambda} \), the total payoff from consuming all \( \lambda \) of \( X \) is larger than the total payoff from consuming incrementally or waiting for pie \( Y \) to arrive. Hence, in any equilibrium, at states \( \lambda X \) with \( \lambda \leq \hat{\lambda} \) the proposer will only make offers over all \( \lambda \) of \( X \). Moreover, this fact together with an argument similar to that used in Lemma 3 implies that the total payoff from consuming all \( \lambda \) of pie \( X \) in state \( \lambda X \) with \( \lambda > \hat{\lambda} \) is strictly greater than the total payoff from consuming incrementally or waiting for pie \( Y \) to arrive. Therefore, at states \( \lambda X \) with \( \lambda > \hat{\lambda} \) the proposer will again make offers over all \( \lambda \).

Because players only make offers over all of what is left of \( X \), one can again find bounds for \( v_i(j, \lambda X) \) and \( \bar{v}_i(j, \lambda X) \) as in Lemma 4. Then one can use an argument similar to the one in Lemma 5 to establish the uniqueness of equilibrium payoffs and to show that these payoffs can only be supported by a strategy profile in which players come to an immediate agreement over \( X \). ■

To complete the proof of Theorem 2, we describe a stationary strategy profile that is an equilibrium. For states \( \lambda XY \), we have already characterized stationary equilibrium offers in (A9) and (A10); player \( i \)'s acceptance rule is then to accept an offer if and only if by accepting he receives at least \( \delta w_i(\lambda XY) \) as we have defined above.
Now consider states $\lambda X$. For each $\lambda \in [0,1]$ let $v_i(j, \lambda X)$, $i,j = 1,2$ be the (unique) solution to the system of equations given by (i)-(vi) in Lemma 4 (with equalities instead of inequalities). In state $(j, \lambda X)$ player $i \neq j$ accepts an offer if and only if accepting gives him a total payoff of at least $\delta (pw_i(\lambda XY) + (1-p)w_i(\lambda X))$.

If the state is $(1, \lambda X)$ and $p > \hat{p}(\delta, r_1, r_2)$ then player 1 offers $\{\lambda, 0\}$ when $\lambda \leq \hat{\lambda}$ and $\{\lambda - \hat{\lambda} - x_2, x_2\}$ such that $x_2$ is the (unique) solution to $rx_2 + w_2(\hat{\lambda}X) = \delta(pw_2(\lambda XY) + (1-p)w_2(\lambda X))$ when $\lambda > \hat{\lambda}$. If $p \leq \hat{p}(\delta, r_1, r_2)$ he offers $\{\lambda - x_2, x_2\}$ such that $x_2$ solves $rx_2 + w_2(\emptyset) = \delta(pw_2(\lambda XY) + (1-p)w_2(\lambda X))$. If the state is $(2, \lambda X)$ and $p > \hat{p}(\delta, r_1, r_2)$ then player 2 offers $\{0, \lambda\}$ when $\lambda \leq \hat{\lambda}$ and $\{x_1, \lambda - \hat{\lambda} - x_1\}$ such that $x_1$ solves $x_1 + w_1(\lambda X) = \delta(pw_1(\lambda XY) + (1-p)w_1(\lambda X))$ when $\lambda > \hat{\lambda}$. If $p \leq \hat{p}(\delta, r_1, r_2)$ he offers $\{x_1, \lambda - x_1\}$ such that $x_1$ solves $rx_1 + w_2(\emptyset) = \delta(pw_1(\lambda XY) + (1-p)w_1(\lambda X))$.

Given this strategy profile no player has a profitable one-shot deviation. Therefore, this strategy profile is indeed an equilibrium.
References


